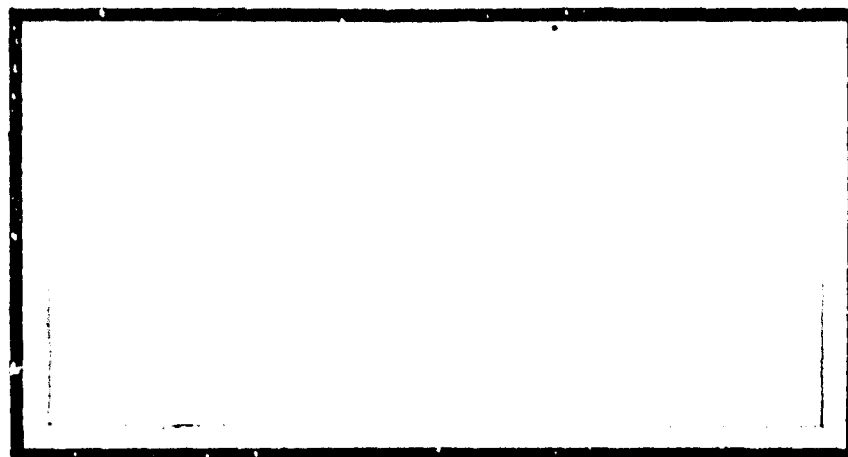


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A STUDY OF FLYING HOURS AND SORTIES AS
PREDICTORS OF B-52H ENGINE FAILURES

Captain Charles L. Dow
Captain Walter L. Schnee

SLSR-6-72A

A STUDY OF FLYING HOURS AND SORTIES AS
PREDICTORS OF B-52H ENGINE FAILURES

A Thesis

Presented to the Faculty of the
School of Systems and Logistics
of the Air Force Institute of Technology
Air University

In Partial Fulfillment of the Requirements for the
Degree of Master of Science in Logistics Management

By

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and

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Chapter 1

INTRODUCTION

Background

The United States Air Force has over six billion dollars invested in jet aircraft engines. (22)¹ To maintain these engines additional investments are made in highly skilled labor and complex equipment. The engines are easily damaged and time consuming to repair. Although any one of these investments could be studied for efficiency and cost control, it seems only logical that the primary way to minimize jet aircraft engine costs is to have only the required number on hand. Basic to the establishment and effective management of this "required" inventory is an accurate technique of predicting future requirements. Accuracy in this forecasting technique will not only yield a monetary savings, but will also enhance the operational capability and effectiveness of the Air Force weapon systems.

The current Air Force methodology for computing jet engine spare requirements is based upon the actuarial forecasting concept. This program consists of,

¹The first number refers to the Bibliography reference number, the second refers to page number(s); e.g., (20:8).

. . . the development and use of actuarial mathematics and the theory of probability for determination of failure rates and the life expectancy for jet aircraft engines. (5:1-2)

The application of these actuarial principles to jet engine demand forecasting is based upon the assumption that failures of engines are a function of age. This age is measured in terms of aircraft flying hours. AFM 400-1, Volume III, and T.O. 00-25-128 explain in detail the Air Force actuarial forecasting system. Air Force Logistics Command (AFLC) managers have frequently questioned the validity of this forecasting method. Actual failures of jet engines have varied widely from AFLC predictions. Consequently, current management feels that in using accumulated flying hours as the sole demand prediction tool of engine failures, they may be neglecting other critical factors.

(22) Another subject of major concern in the utilization of the actuarial forecasting technique is that it draws upon past data and does not provide for the input of variables based upon expected future states of nature such as sorties. (20:8)

Considerable effort is being expended by many government researchers in these two areas. This thesis limits its study to the consideration of flying hours and sorties as feasible predictors of engine failures. Historical records and estimates of future flying programs will be the basic sources of data.

The Problem

The number of spare aircraft engines required for a specific model of aircraft in the Air Force inventory is currently determined as a function of flying hours. Air Force Logistics Command managers suspect that the sole use of flying hours as a demand prediction tool does not yield an accurate picture of engine demands. A major portion of that demand is premature engine failures which are also currently predicted on the basis of flying hours. If an accurate forecast of premature engine failures can be made, the inventory manager can then make a much improved estimate of the resources he will have to expend in support of aircraft propulsion units. The specific problem, therefore, may be phrased as the question: Are there other aircraft program activities which can be used as demand prediction tools to provide a more accurate estimate of engine failures?

Assumptions and Limitations

The authors believe that certain basic assumptions must be made before a program activity can be used as a demand prediction tool. These assumptions are as follows:

1. The future of a program activity (such as flying hours or sorties) can be accurately forecast.
2. A reliable, measurable relationship exists between the demand element and the program activity.
3. The data generated by the program activities are accurate.

There were three major limitations placed upon this study. They were:

1. Only one aircraft-engine combination was to be used for study.
2. A minimum of three years' data was to be collected for analysis.
3. The selection of program activities was limited to those activities currently being measured by quantitative techniques prescribed in Air Force directives.

Objectives

The objectives of this thesis were threefold:

1. Identify program activities that may be suitable for use as engine failure prediction tools.
2. Develop a failure predicting model with regression analysis techniques.
3. Statistically test and evaluate the developed model.

Hypothesis

The research methodology utilized in this thesis was designed to test the following hypothesis: A combination of flying hours and sorties can be utilized to yield accurate jet aircraft engine failure forecasts.

Overview

This thesis attempts to show that engine failures are dependent upon some combination of historical flying

hour and sortie data. A method of arrangement was undertaken which transformed the collected data into specific historical groupings. These specific groups of data were then statistically analyzed and a forecasting model developed. The statistical analysis was performed by application of multiple correlation and regression techniques to the data. The Biomedical Series, BMD02R, Stepwise Multiple Regression package (10) was chosen for use because of its completeness in data output and apparent versatility in use.

Chapter 2 sets forth how the data was collected, screened and prepared for use in the development of a set of predictive models. The third chapter delineates in detail the model development, the test of statistical assumptions, and the evaluation of predictive power. Chapter 4 contains the interpretation of model behavior, conclusions concerning the effectiveness of the model, and recommendations for further study.

Chapter 2

DATA COLLECTION AND ARRANGEMENT

This chapter will describe how the data was collected, screened, and prepared for use in the development of a set of predictive models. It is broken into six topics: (1) Selection of Program Activities, (2) Selection of Aircraft-Engine Combination, (3) Data Collection, (4) Data Screening, (5) Preliminary Data Study, and (6) Data Arrangement. Model development, verification, and validation are discussed in the succeeding chapter.

Selection of Program Activities

Flying hours and sorties were the program activities chosen to be included in this study of jet aircraft engine failures. Design, operational and logistical personnel have long considered accumulated flying hours as a measure of aircraft engine life. Although useful, this measure unfortunately does not produce the accuracy in predicting engine failures desired by AFLC.

Engine design personnel seem to have concluded that frequent short durations of extreme temperatures have a detrimental effect on the life of a jet engine. This condition is normally generated by demanding maximum thrust from the engine. The running of an engine into and out of

this critical temperature range is defined as a cycle. At present, engine cycles are not recorded in any manner.

However, sorties are recorded and if it can be assumed that a sortie contains at least one cycle, generated at take-off, then there exists an imperfect, but possibly useful, measure of cycles that could be applied to the explanation of engine failures. This reasoning coupled with the idea that simple frequency of use may shorten engine life seemed to provide a sufficient argument for the consideration of sorties as an element of engine failures. Finally, both of these activities are readily understood, easily measured, and currently recorded.

Flying hours. Flying hours are defined as "all time of flight of a military aircraft creditable to the aircraft, its equipment and personnel aboard." (4:100) Currently all engine forecasts are based on the number of hours an engine type has flown. This historical approach has largely neglected to consider the frequency of use and the non-flying operating time accumulated on an engine. For example, aircraft sorties, taxi time, and maintenance test runs are recorded, but are not considered as a part of the USAF actuarial forecasting system. Flying hours should be considered as only a part of the engine failure problem, not all of it.

Sorties. A sortie is defined as,

A flight, by the same aircraft, intended to accomplish a specific assigned task. A sortie is normally terminated by engine shutdown. A flight beginning and ending at the same airdrome, is

considered one sortie, even though touch-and-go landings may be accomplished at other airdromes. (4:209).

Engine managers, engineers, and using Air Force commands have often expressed the opinion that sorties may be a major factor in the life of an engine. (22) A preliminary study was performed by the operations research personnel at AFLC Headquarters in 1970 under the assumption that sorties could be related to engine failures. The early results of this study showed promise for using a combination of sorties and flying hours as an engine failure prediction tool. (23)

A definitive RAND Corporation study (RM-6010-PR, June 1969) reported the results of simulation exercises in this general area. They also indicated that a combination of sorties and flying hours should result in a superior prediction tool.

Selection of Aircraft- Engine Combination

Several aircraft-engine combinations were considered for use in this study. Each was evaluated against the following criteria:

1. Were there enough observations available so that a significant statistical analysis could be applied?
2. Did the aircraft represent a major weapon system of the Air Force inventory?
3. Was the aircraft considered as having a future in the Air Force inventory?

4. Did the aircraft represent a major investment in Department of Defense resources?

5. Were there at least three years' of unclassified program activity data available?

The final selection consisted of the B-52H strategic bomber and its TF33-3 engines. The Air Force possesses 99 of these aircraft. There are 792 installed engines and 96 spares in the supply system. (22) The selection of this aircraft-engine combination was thoroughly discussed with engine management personnel at AFLC Headquarters. They expressed the opinion that it was highly suited to the type of study being performed.

Data Collection

The following data was collected on the entire B-52H fleet:

1. Total number of flying hours accumulated per month.
2. Total number of sorties accumulated per month.
3. Total number of engine failures requiring depot or intermediate level maintenance accumulated per month.

All this data was available within the Directorate of Propulsion and Auxiliary Power Systems Office, Headquarters AFLC.

To simplify the explanation of the data collection technique, Table 1 was constructed.

Table 1
DATA COLLECTION TECHNIQUE

DATA	SOURCE OFFICE	SOURCE DOCUMENT	EXTRACTION METHOD
Flying Hours Accumulated	AFLC/MMAPP	Monthly Aerospace Vehicle Status Report G033BNF	Manual
Sorties Accumulated	AFLC/MMAPP	Monthly Aerospace Vehicle Status Report G033BNF	Manual
Actual Recorded Failures	AFLC/MMPP	Aircraft Engine Removal and Loss Report D024FI03-N1	Manual

Cost and time constraints precluded a mechanized data gathering technique.

The time frame over which the data was collected was 1 October 1965 through 30 September 1971, a period consisting of 72 months. Flying hour and sortie data were available for the entire 72-month period. However, engine failure data was available only from 1 July 1968 through 30 September 1971, a period of 39 months. Figure 1 graphically presents the time periods covered by the data.

Data Screening

After the basic data of engine failures, flying hours, and sorties for the B-52H aircraft fleet were collected, a

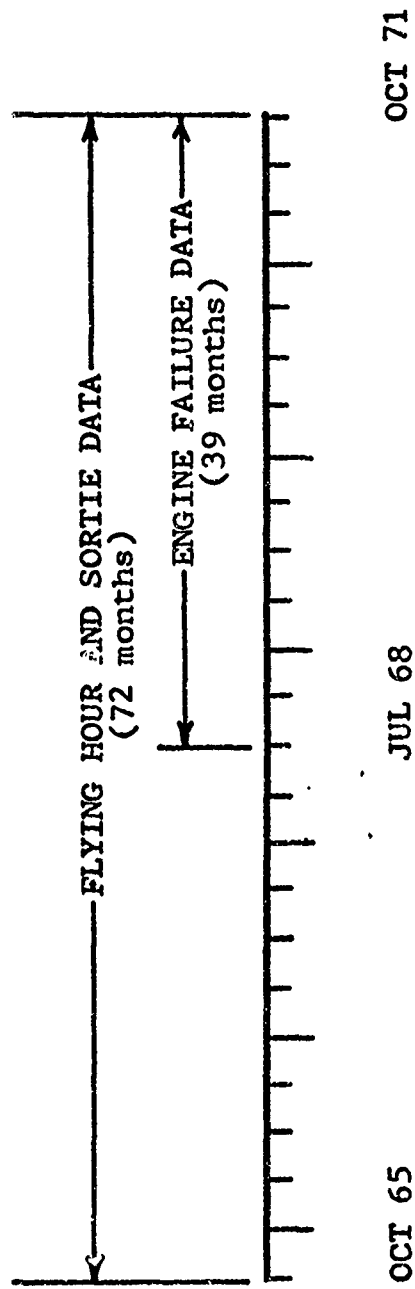


Figure 1
Time Frame of Available Data

process of screening and organizing was employed to construct the Master Data Sheet, Table 2.

The data extracted as engine failures was examined for (1) invalid reason for engine removal codes, (2) engine removal codes that related to other than depot or field level failures, and (3) general errors in keypunch entry. Each engine removal was a separate entry in the D024FI03-N1 report; therefore, it was necessary to group the failures by calendar month periods for inclusion on the Master Data Sheet. The examination of the removal codes and the grouping of the data was mechanized on the G.E. 115 Batch Remote Computer.

The flying hour and sortie data was examined for (1) missing monthly entries and (2) general errors in keypunch. The data was extracted from the G033BNF report, then manually examined, and entered directly onto the Master Data Sheet.

Preliminary Data Study

As the first step in the preliminary study, a set of histograms and time series graphs were prepared for each of the three basic variables measured. Figures 2a, 2b, 3a, 3b, 4a, and 4b depict this part of the study. Next, a set of curves were fit to specific groupings of the data using the subroutine "CURFIT" available on the G.E. 615 series computer time sharing system and adapted to cathode ray tube (CRT) display. This program fits six different least squares

(Text continues on page 21)

Table 2

Master Data Sheet

		Observa. Number	Date	Observed Failures	Observed Flying Hours	Observed Sorties
FLYING HOUR AND SORTIE DATA	ENG. FAILURE DATA MODEL BASE PERIOD	1	Oct 65		6172	528
		2	Nov		4688	502
		3	Dec		3108	341
		4	Jan 66		5132	429
		5	Feb		3998	368
		6	Mar		4665	424
		7	Apr		4253	457
		8	May		4631	478
		9	Jun		4636	510
		10	Jul		3794	415
		11	Aug		4408	479
		12	Sep		3968	456
		13	Oct		4562	511
		14	Nov		3503	406
		15	Dec		3148	422
		16	Jan 67		4144	454
		17	Feb		4140	449
		18	Mar		4710	506
		19	Apr		4948	490
		20	May		4732	462
		21	Jun		4072	438
		22	Jul		4002	423
		23	Aug		4104	420
		24	Sep		3396	354
		25	Oct		4261	396
		26	Nov		4497	447
		27	Dec		2815	314
		28	Jan 68		3509	387
		29	Feb		5199	535
		30	Mar		4706	475
		31	Apr		4473	460
		32	May		3422	425
		33	Jun		3559	459
		34	Jul	44	3696	493
		35	Aug	37	3405	399
		36	Sep	37	3863	423
		37	Oct	44	4064	474
		38	Nov	34	3563	390
		39	Dec	27	2901	362
		40	Jan 69	44	3031	406
		41	Feb	28	3429	367
		42	Mar	19	3386	372
		43	Apr	32	3426	414

Table 2 (continued)

			Observa. Number	Date	Observed Failures	Observed Flying Hours	Observed Sorties
FLYING HOUR AND SORTIE DATA	ENGINE FAILURE DATA	MODEL BASE PERIOD	44	May 69	26	3120	383
			45	Jun	10	2559	333
			46	Jul	25	2727	322
			47	Aug	35	3230	362
			48	Sep	25	2642	331
			49	Oct	20	1401	198
			50	Nov	23	2604	320
			51	Dec	28	2604	330
			52	Jan 70	12	2598	313
			53	Feb	23	2877	318
			54	Mar	23	2381	302
			55	Apr	27	2266	306
			56	May	36	2854	327
			57	Jun	29	2137	433
			58	Jul	33	2817	377
			59	Aug	33	2699	332
	PREDICTION PERIOD		60	Sep	21	2704	330
			61	Oct	21	2993	367
			62	Nov	11	2588	333
			63	Dec	20	2486	327
			64	Jan 71	18	2096	305
			65	Feb	21	3034	335
			66	Mar	33	3167	392
			67	Apr	29	3144	421
			68	May	18	2870	383
			69	Jun	34	2921	420
			70	Jul	24	3053	427
			71	Aug	25	3218	430
			72	Sep	18	3008	403

NOTE: Failure Data Not Available Before July 1968.

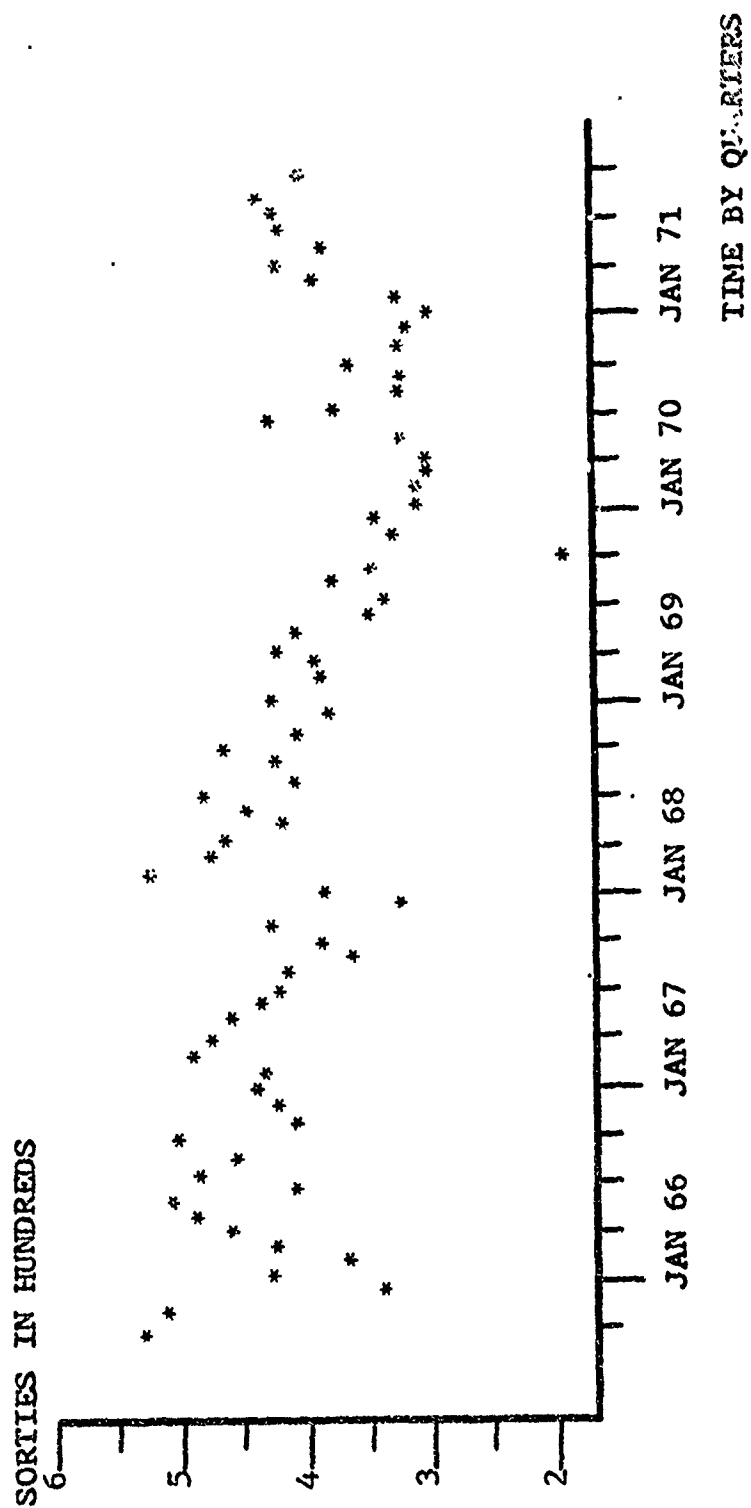


Figure 2a
B-52H Monthly Sortie Generation History

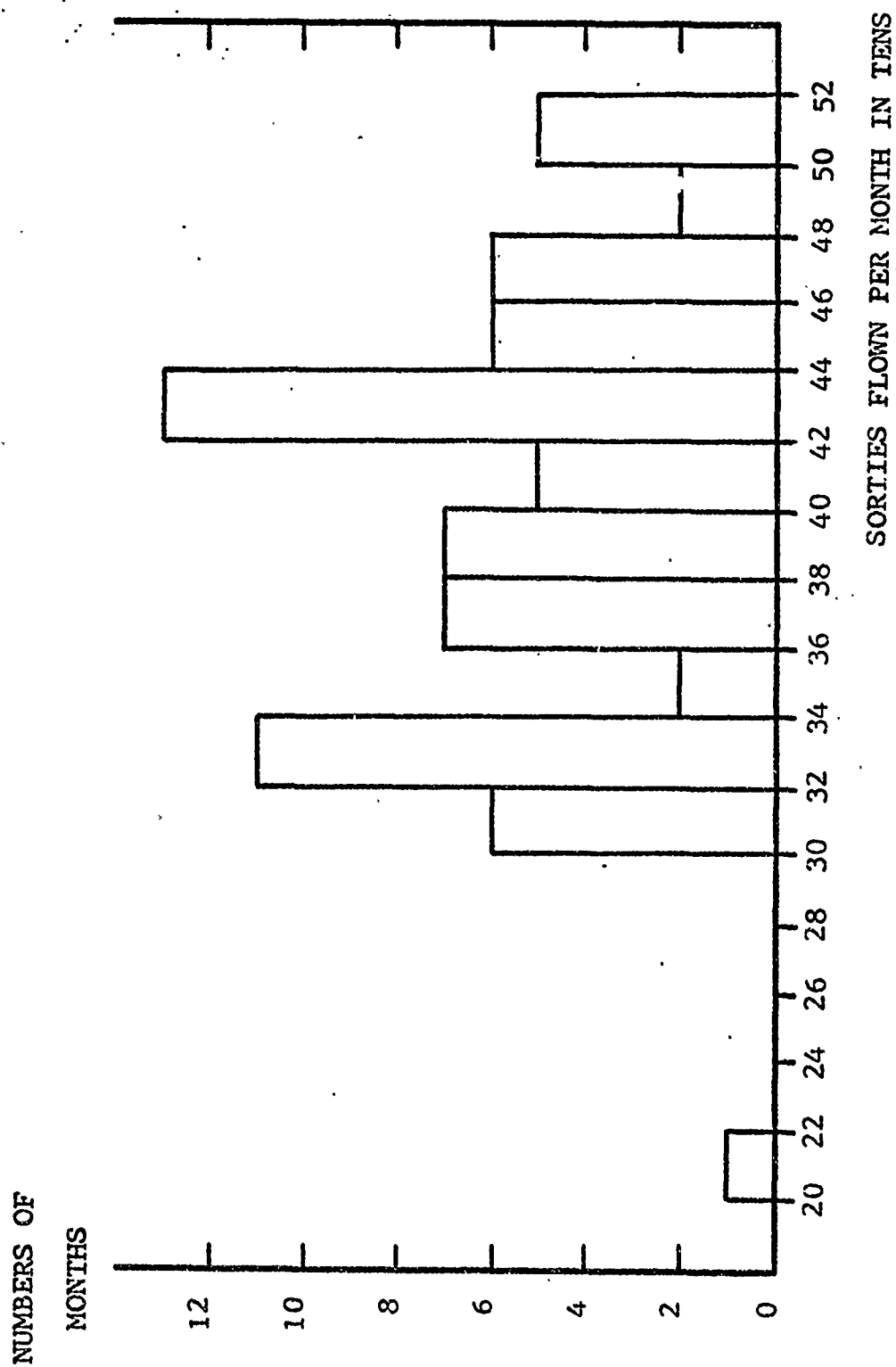


Figure 2b
Histogram of Monthly Sortie Data

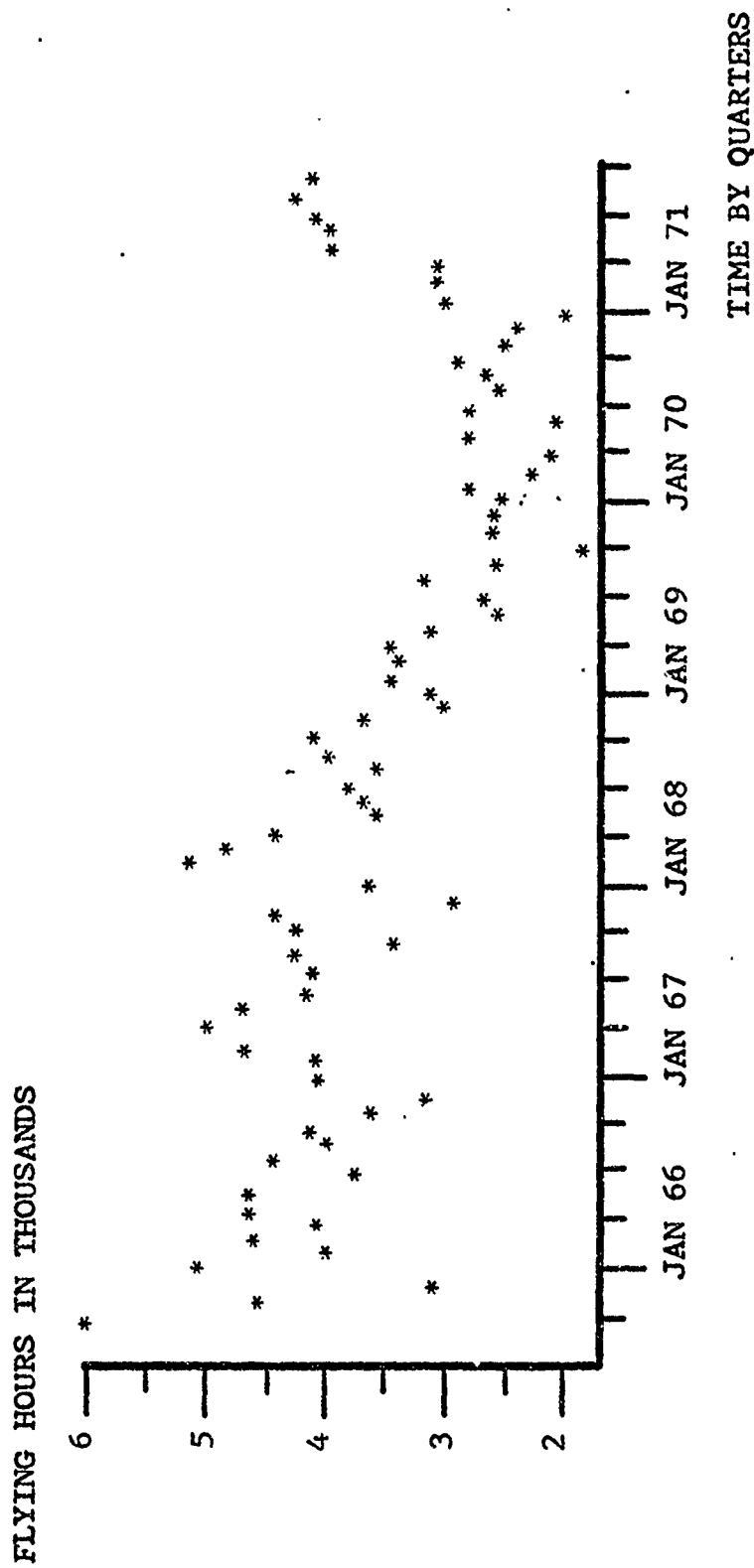


Figure 3a
B-52H Monthly Flying Hour History

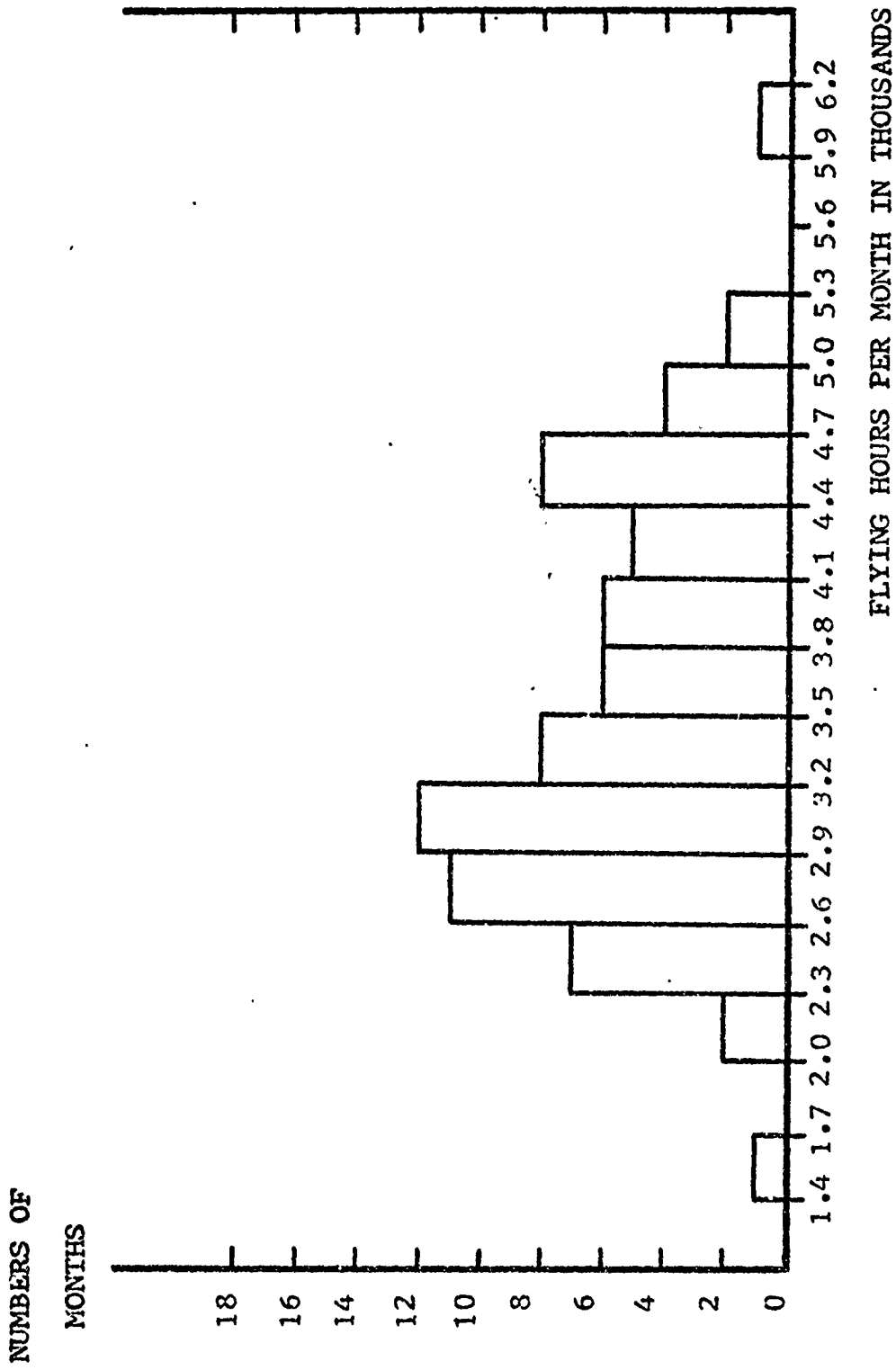


Figure 3b
Histogram of Monthly Flying Hour Data

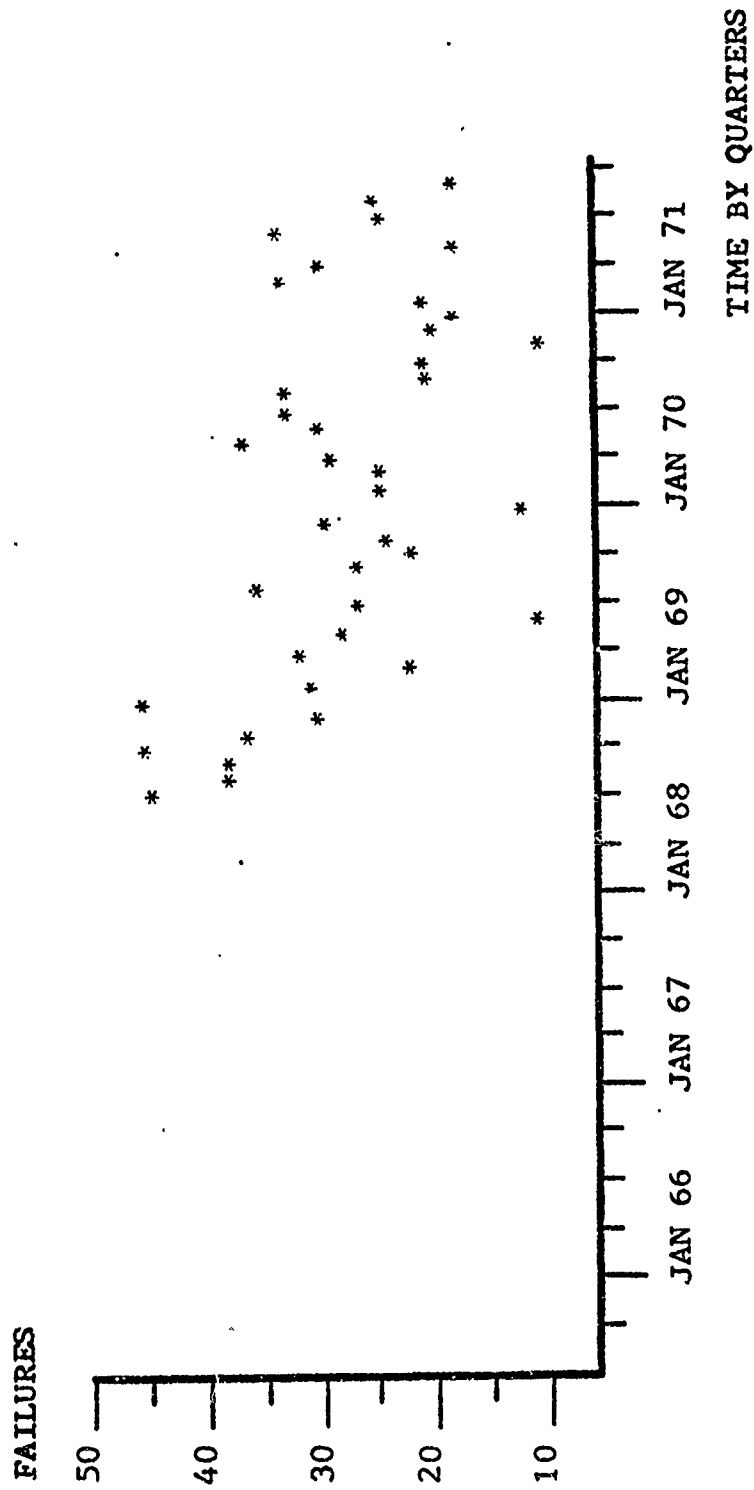


Figure 4a
TF33-3 Engine Failure History

NOTE: Failure Data Not Available Before July 1968.

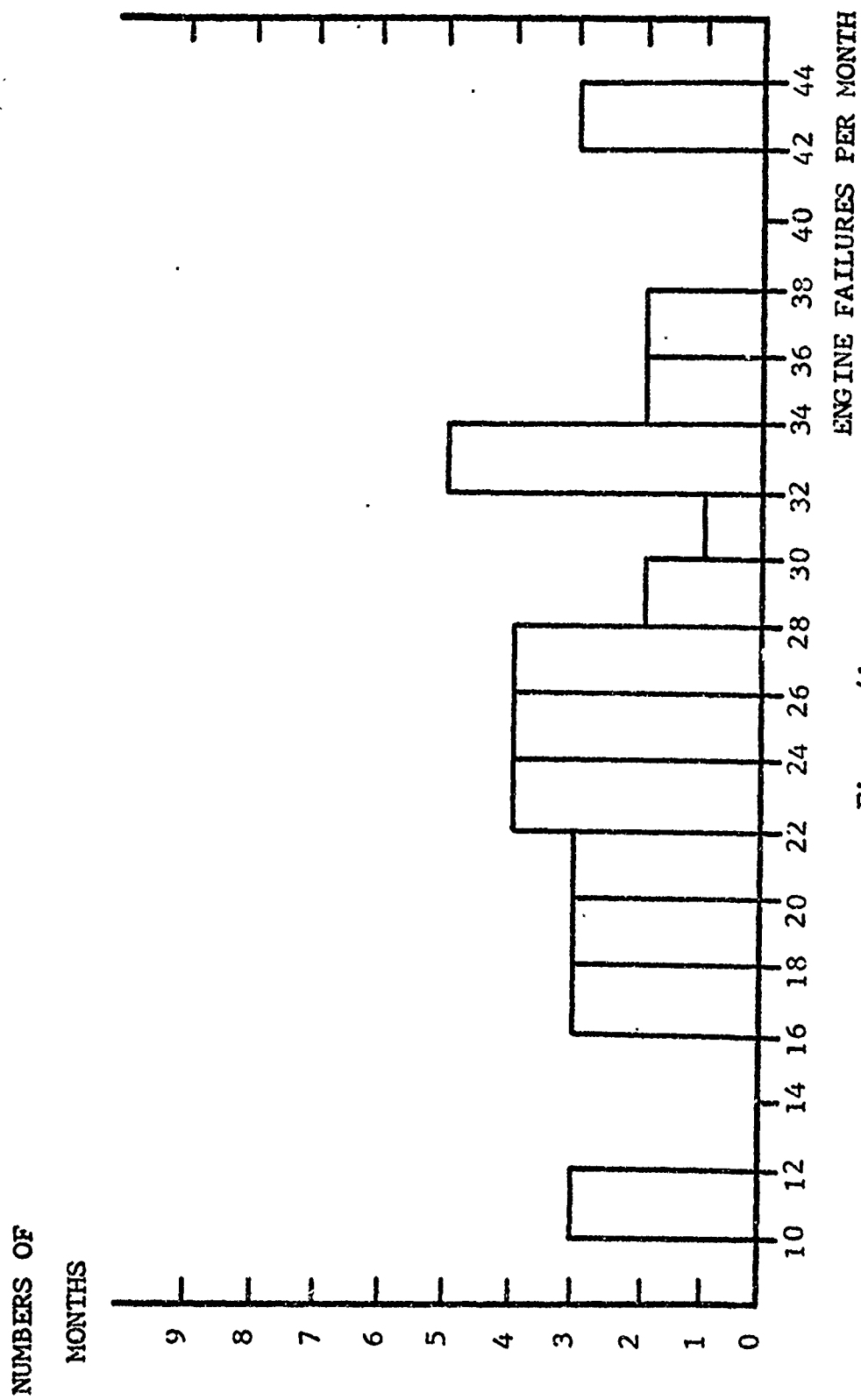


Figure 4b

Histogram of Engine Failure Data

NOTE: Data available July 1968 - September 1971 only

curves to the supplied data. Six coefficients of determination (R^2) are then presented so that the operator may select the curve he wishes to plot. The data groupings were (1) failures versus sorties, (2) failures versus flying hours, and (3) sorties versus flying hours. In the first and second groupings a linear function was determined to be the best curve fit; but, in both cases, it was far from what might be called a "good fit." Reference Figures 5 and 6. These observations gave rise to a suspicion that there may develop a relationship other than linear when different variables were combined for the multiple regression analysis approach. However, none developed or was found to exist. The fit of a hyperbolic function to the third grouping, sorties versus flying hours, Figure 7, helped substantiate the authors' belief that while sorties and flying hours are related, the relationship tended to be curvilinear as opposed to linear and is not one of extremely high correlation. Thus, sorties are probably not totally dependent nor independent of flying hours, and may be an additional factor worthy of consideration when attempting to explain engine failures.

Data Arrangement

The data arrangement process was broken down into three steps: (1) determination of prediction period, (2) historical grouping of the data, and (3) adaption to the Biomedical Program's standard matrix form.

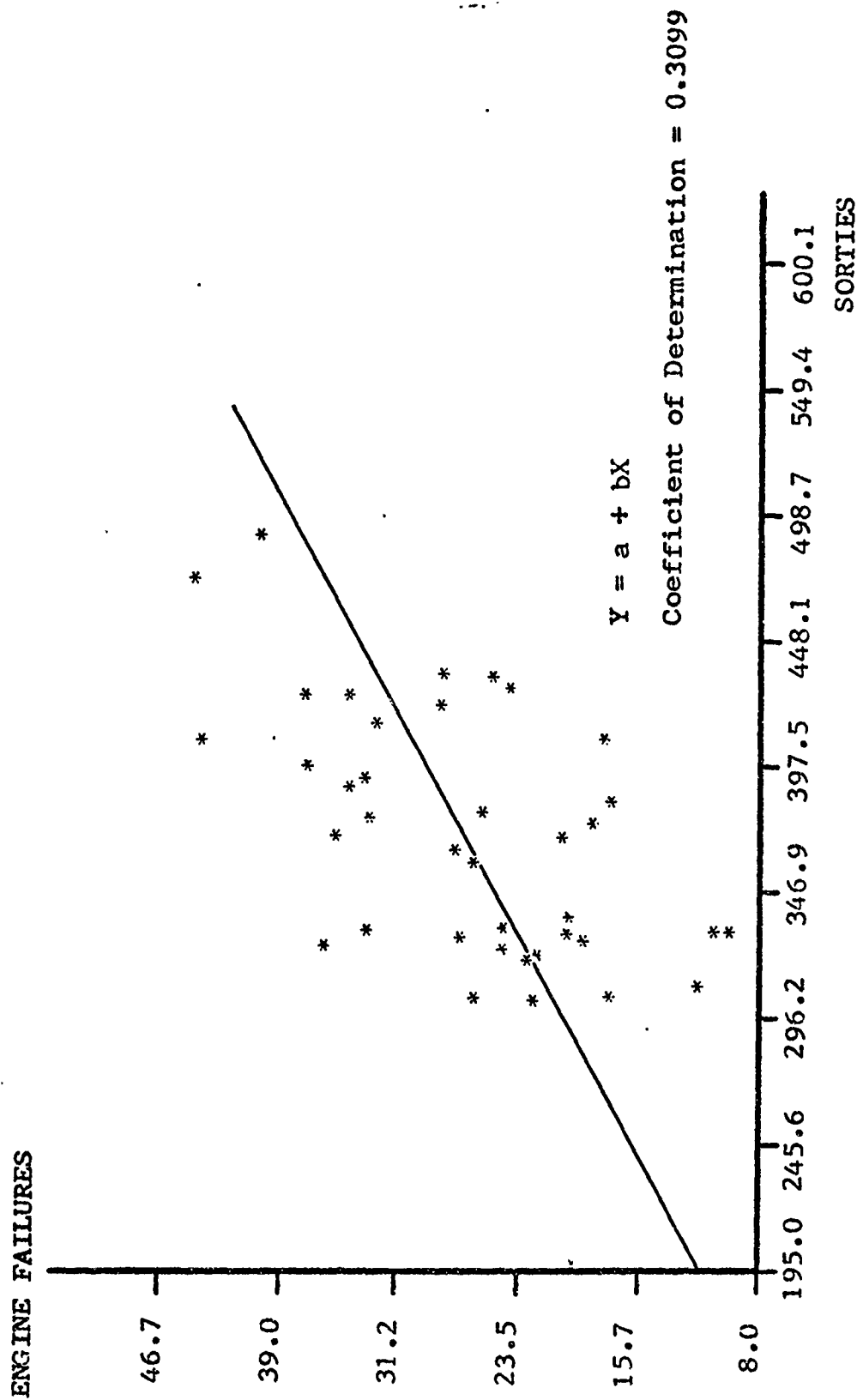
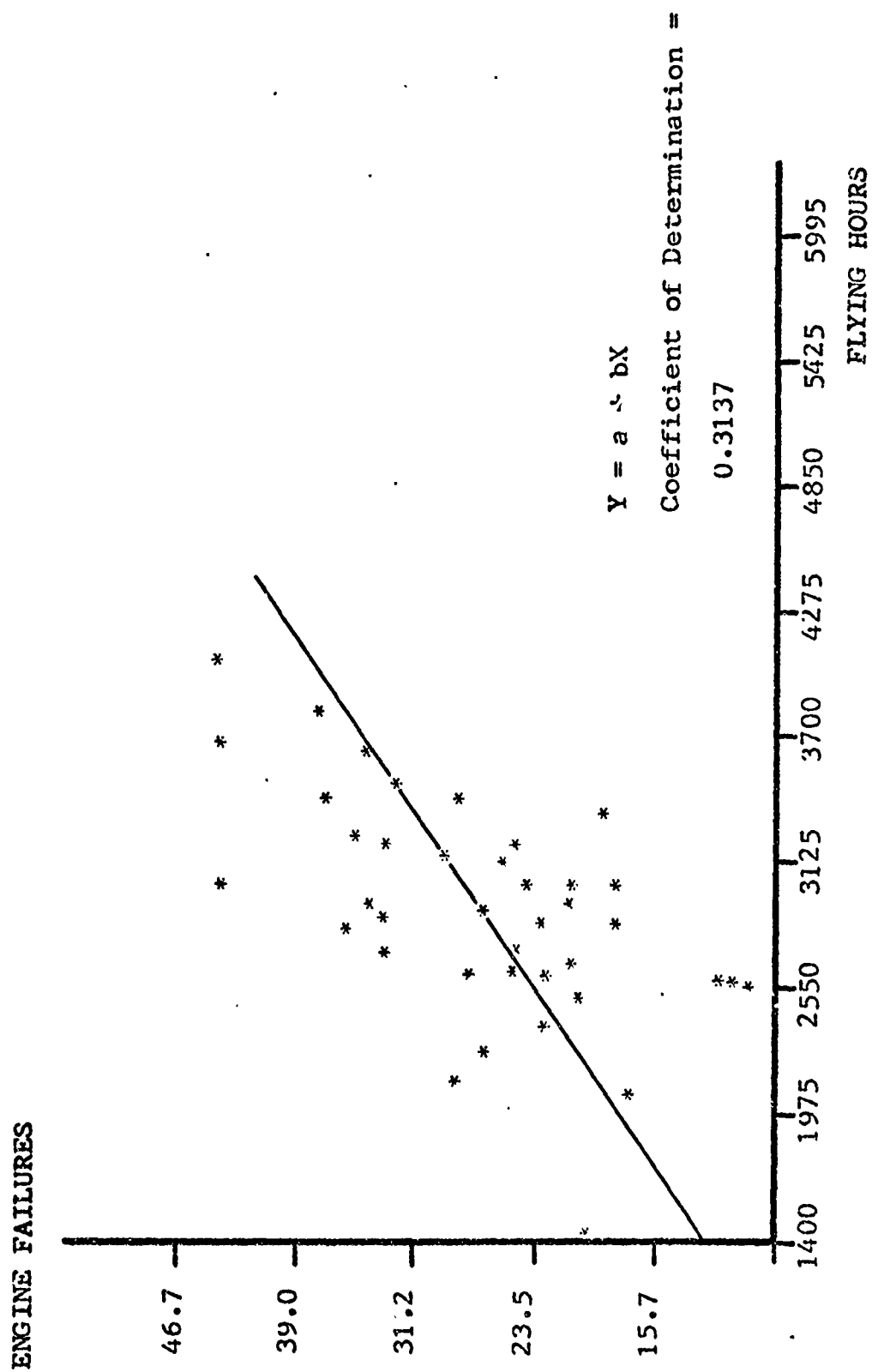


Figure 5

Plot of Engine Failures vs Sorties



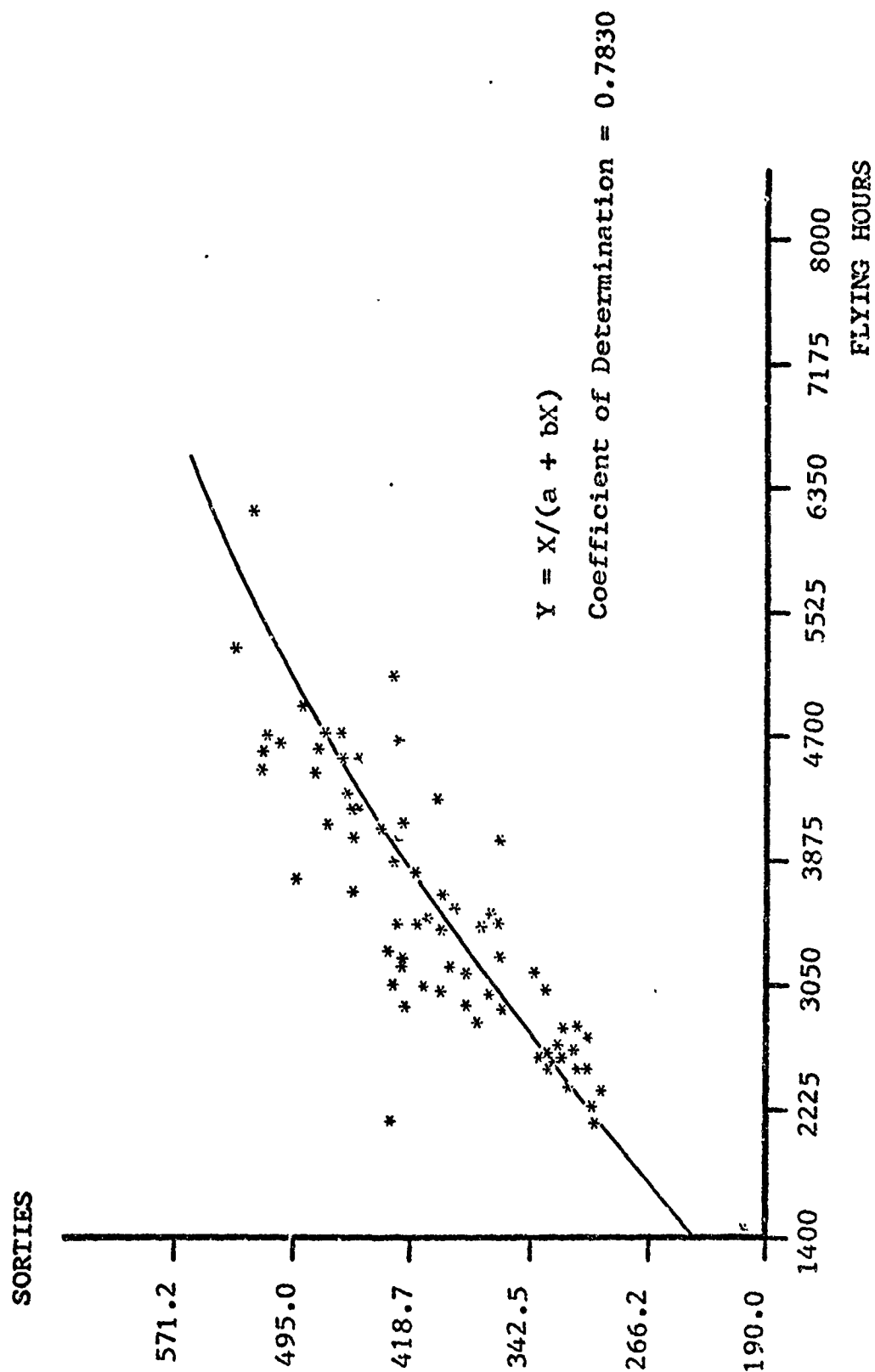


Figure 7
Plot of Sorties vs Flying Hours

Prediction period construction. Prior to the historical grouping of the data, it was necessary to determine what the prediction period was to be so that the data could be structured as portrayed in Figure 8. Using this prediction period, two techniques were designed to validate the forecasting ability of each of the multiple regression developed models.

1. Technique 1 was the application of the model derived from the base period 1 July 1968 through 30 September 1970 to each of the eleven months in the prediction period, 1 November 1970 through 30 September 1971. The same linear model was employed for all eleven predictions. Reference Figure 8.

2. Technique 2 applied the linear model developed from the base period 1 July 1968 through 30 September 1970 to the prediction of November 1970 failures. Then the linear equation was changed by adding October 1970's observed failures, flying hours and sorties to the data base, subtracting the oldest set of observations, July 1968, and recomputing the regression. In this manner, the most current data was utilized and the data base maintained at 27 months. There were eleven different equations developed, one for each month in the prediction period.

In general, for both techniques all forecasts were computed on the first day of each month for all engine failures that would occur during the following month. For example, on 1 October 1970, a prediction of engine failures

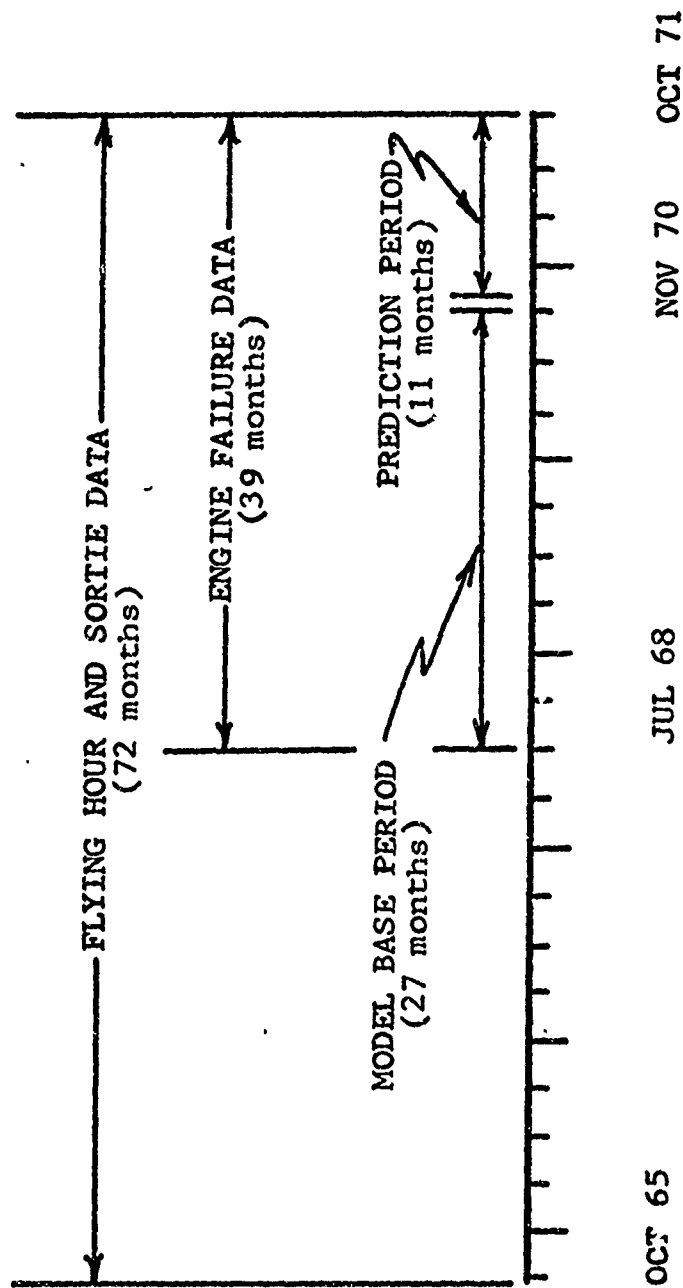


Figure 8
Graphical Illustration of Prediction
Period Construction

for the entire month of November was made. This approach made necessary the estimation of flying hour and sortie totals of the months of October and November.

Historical grouping. It was highly conceivable that engine failures were the result of some historical combination of flying hours and sorties. The question, though, was how were failures, flying hours, and sorties related; and if a useful relationship existed, was it time-oriented in the historical context? The stepwise multiple regression package selected was designed to relate independent and dependent variables and posed no particular problem. However, the historical orientation was more difficult to handle. The idea presented here was to approach the problem by "creating" variables of historically grouped flying hour and sortie data. Perhaps the easiest way to describe this grouping process is to define the variables used. Specifically let,

X_{i1} = the observed engine failures for the i th month. ($i = 34, 35, 36, \dots, 72$)

X_{i2} = the observed sorties flown for the i th month. ($i = 1, 2, 3, \dots, 72$)

X_{i3} = the observed flying hours flown for the i th month. ($i = 1, 2, 3, \dots, 72$)

These three variables are the original data collected and tabulated in Table 2.

In general, let,

X_{ij} = the j th independent variable (accumulated sorties) associated with the i th month,
 $(j = 2, 4, 6, \dots, 66)$
 $(i = 34, 35, 36, \dots, 72)$

X_{ir} = the r th independent variable (accumulated flying hours) associated with the i th month.
 $(r = 3, 5, 7, \dots, 67)$
 $(i = 34, 35, 36, \dots, 72)$

These are the created variables and were determined by the use of the following equations:

$$X_{ij} = \sum_{q=i}^{i-m+1} X_{q2} \quad \begin{array}{l} \text{Sortie Accumulation} \\ \text{where } j = 2m \end{array}$$

$$X_{ir} = \sum_{q=i}^{i-m+1} X_{q3} \quad \begin{array}{l} \text{Flying Hour Accumulation} \\ \text{where } r = 2m + 1 \end{array}$$

where,

m = the number of months of historical data to be summed. ($m \leq 33$)

In this manner, 64 independent variables were "created" to show historical accumulation of flying hour and sortie data. Thirty-two of those variables, ranging from two months to 33 months accumulated data, pertain to sortie history. The remaining 32 variables are similar accumulations of flying hour history. After the created variables are combined with the two observed variables, there are 66 independent variables available for regression against the

dependent variable, engine failures. Table 3 is the key to variable identification.

Table 3
VARIABLE IDENTIFICATION KEY

If variable is:	Then it is defined as:
X_{i1}	Observed engine failures for the i th month
X_{ij} , where $j = 2, 4, 6, \dots, 66$	$j/2$ months historical accumulation of total fleet sorties
X_{ir} , where $r = 3, 5, 7, \dots, 67$	$(r-1)/2$ months historical accumulation of total fleet sorties

NOTE: i equates to an observation as presented in Table 2. In this table, $i = 34, 35, 36, \dots, 72$.

Biomedical Standard Matrix. In general, the Biomedical programs require that all data be prepared in a two-dimensional matrix that is arranged by case, by variable. (10:11) A case equates to a month's observations in this instance. This specific matrix was built with punch cards and stored in a permanent file on the G.E. 615 computer. The next major step was the development of the prediction models using the Biomedical Series Program BMD02R, Stepwise Multiple Regression package.

Chapter 3

MODEL DEVELOPMENT

This chapter delineates in detail how the model was developed, the statistical assumptions verified, and the predictive power of each model validated. It is divided into four topics: (1) Computer Program Description, (2) Building the Model, (3) Verification of the Statistical Tool, and (4) Validation of the Model. The analysis of the obtained results and the conclusions drawn from the analysis are discussed in the succeeding chapter.

Program Description

The BMD02R stepwise multiple regression program computes a series of linear regression equations in a prescribed sequential manner. (10:233) The first step selects the independent variable that explains the greatest part of the variation in the dependent variable and calculates the simple regression relationship which exists between them. The second independent variable is selected on the basis of making the greatest additional contribution to the explained variation. Continuing in this manner, the program carries out the regression for each independent variable that can significantly contribute to the reduction of unexplained variation in the dependent variable. At each step, the

coefficient of multiple correlation (R), the standard error of the estimate, and an analysis of variance on the regression and residual values are presented. Also included at each step is a listing of the constant term and all variables entered into the equation along with their respective net regression coefficients and standard errors. The final portion of the output at each step is a listing of all of the independent variables not included in the equation and their respective partial correlation coefficients. These coefficients give an indication of the relative importance of each of the variables not yet entered into the regression equation. After the last step, a listing of the residuals is prepared. These residuals are the variation in the dependent variable not explained by the multiple regression equation of the last step.

Optional output features available are: (1) a mean and standard deviation table for all variables, (2) a covariance matrix, (3) a correlation matrix, (4) a summary table, and (5) graphic plots of the residuals against selected independent variables that appear in the final regression equation. (10:233)

Building the Model

In Chapter 2 the initial model base period was established to be from July 1968 through September 1970, a period of 27 months.¹ The first stepwise regression was run on this selected data with no specified limitation set on the number of steps that the program would execute. A total of 25 steps was taken by the program before it reached the internally specified F test alpha (significance) levels of .01 for variable inclusion and .005 for variable deletion. Careful observation of all of the data presented indicated that the standard error of the estimate, or sample standard deviation of the regression, reached a minimum point at the 20th step. This occurrence is portrayed in Figure 9. With the standard error of estimate at a minimum value of 0.5650 engine failures, and the coefficient of multiple determination (R^2) equal to 0.9990 the regression equation at the 20th step was selected for use in the Technique 1 predictions. The specific equation is listed in Figure 10 because of its length. A regression equation that contains 20 variable coefficients is somewhat dubious; therefore, the population net regression coefficients were tested for significance in two separate manners. (24:788-793)

¹Technique 1 and Technique 2 were virtually identical in all aspects of development and verification; therefore, to maintain simplicity, only Technique 1 is discussed in this chapter. The one exception is that during the validation process the prediction ability of Technique 2 is included in the discussion.

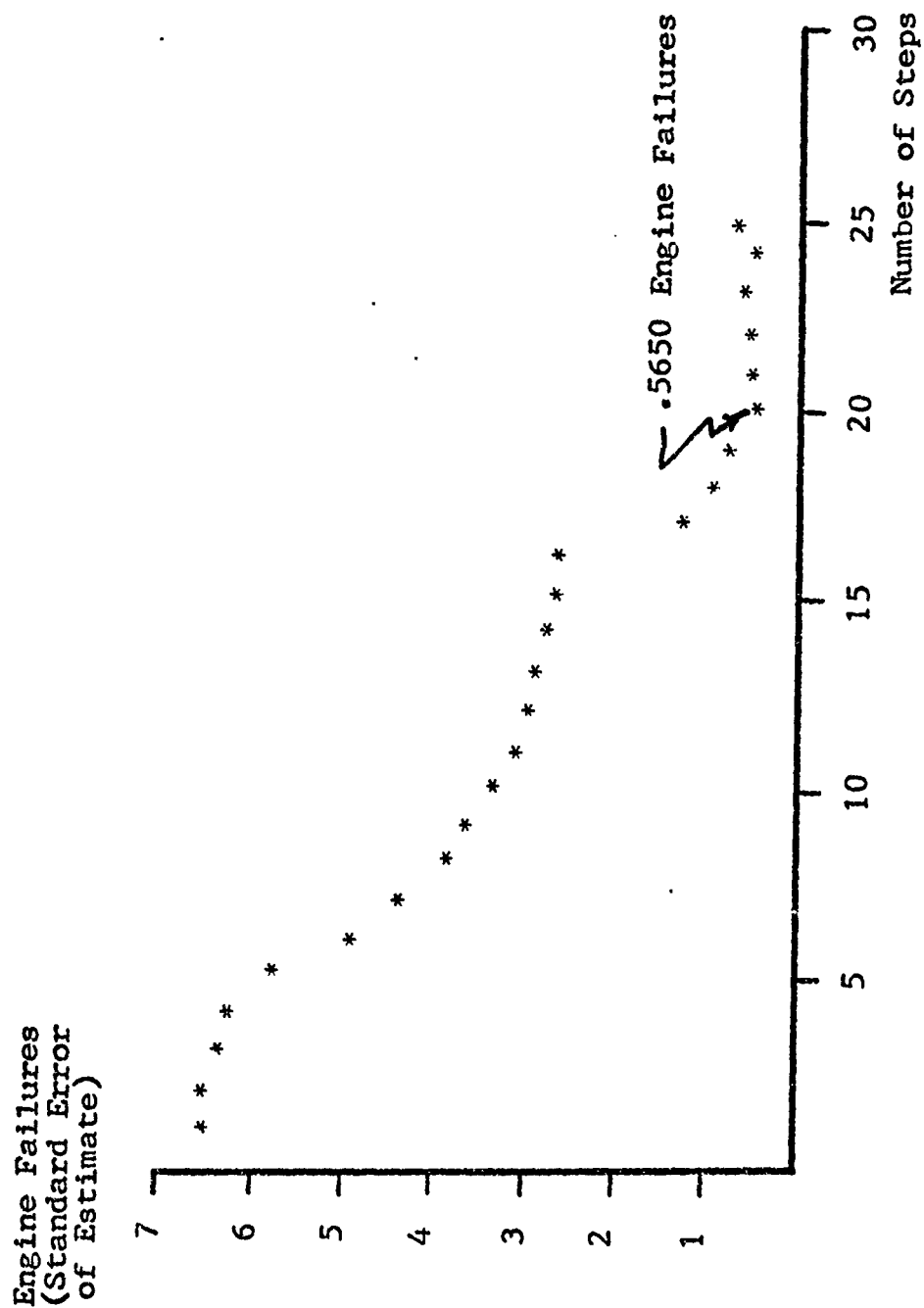


Figure 9
Standard Error of Estimate
By Regression Step

$$Y_i = \left\{ \begin{aligned} &(-148.34809) + \left[-(0.03597 * X_{i2}) + (0.00386 * X_{i3}) + (0.07109 * X_{i4}) - \right. \\ &(0.01457 * X_{i7}) + (0.01387 * X_{i9}) - (0.01286 * X_{i10}) - (0.02684 * X_{i12}) + \\ &(0.00493 * X_{i15}) + (0.04749 * X_{i20}) - (0.00600 * X_{i23}) + (0.00150 * X_{i25}) + \\ &(0.01047 * X_{i28}) + (0.01509 * X_{i32}) - (0.10957 * X_{i34}) - (0.00451 * X_{i37}) + \\ &(0.09315 * X_{i38}) + (0.00086 * X_{i45}) - (0.04043 * X_{i54}) + (0.08306 * X_{i56}) - \\ &\left. (0.02947 * X_{i62}) \right] \end{aligned} \right\}$$

Figure 10

Technique 1 Forecasting Equation

NOTE: Y_i = the dependent variable (or engine failures)

The BMD package program, at each step, tests the hypothesis that the population net regression coefficients (B_i) are zero, in such a manner that it is a test of the overall significance of the regression line. The hypothesis being tested was,

$$H_0 : B_1 = B_2 = \dots B_k = 0$$

$$H_1 : B_1 \neq B_2 \neq \dots B_k \neq 0$$

An F statistic was calculated by performing an analysis of variance of the regression and residual values. The degrees of freedom present for the regression values, at the 20th step, were $p - 1 = 20$ where p is equal to the number of independent variables in the regression equation plus one for the constant term. The degrees of freedom present for the residuals, at the same step, were $n - p = 6$ where n was equal to the total number of observations (or cases) considered. The F critical value with 20 and 6 degrees of freedom at the .05 alpha level equaled 3.86. At step 20 the F statistic was calculated to be 313.882.

It is obvious that the F statistic was significant and the null hypothesis rejected. Technically, it could further be said that there was regression in the population and the improvement brought by fitting this regression plane was not due to chance. It should be noted, though, that each step (20 in all) was successful in rejecting the stated null hypothesis.

At this point, a different tack was taken and the population net regression coefficients were tested separately

for significance with a t test. The hypothesis under test was:

$$H_0 : B_i = 0$$

$$H_1 : B_i \neq 0$$

A t statistic was calculated for each coefficient by dividing that net regression coefficient by its standard error. The degrees of freedom present for each t statistic were $n - p = 6$. At the 20th step, with 6 degrees of freedom, an alpha level of .05, and employing a two-tailed test, $t_{\text{crt}} = \pm 2.447$. Table 4 is a tabulation of the individual t tests and their comparison to t_{crt} .

From Table 4, it can be seen that only one variable, Variable 32, did not reject the null hypothesis. Therefore, it could be said that there were 19 significant variables in the regression equation.

The same F and t tests were applied to the equations developed for Technique 2 with similar results. The F tests failed to reject the stated null hypothesis at any time and the t tests were equally unsuccessful in limiting the number of significant variables. The large number of variables included in the regression equations and the results of the F and t tests raised serious doubts about the validity of the data arrangement being employed.

Verification of the Statistical Tool

The use of multiple regression developed models in making statistical inferences implies that several assumptions

Table 4

"t" TEST FOR SIGNIFICANCE ON
SAMPLE REGRESSION EQUATION

VARIABLE	NET REGRESSION COEFFICIENT (a)	STANDARD ERROR OF NET REGRESSION COEFFICIENT (b)	"t" VALUE (a/b)
2	-0.03597	0.00613	- 5.867
3	0.00386	0.00052	7.423
4	0.07109	0.00567	12.337
7	-0.01457	0.00053	-27.490
9	0.01387	0.00053	26.169
10	-0.01286	0.00515	- 2.497
12	-0.02684	0.00492	- 5.455
15	0.00493	0.00033	14.939
20	0.04749	0.00372	12.766
23	-0.00600	0.00042	-14.285
25	0.00150	0.00030	5.000
28	0.01047	0.00398	2.630
32	0.01509	0.00676	2.232*
34	-0.10957	0.00488	-22.452
37	-0.00451	0.00051	- 8.843
38	0.09315	0.00545	17.091
45	0.00086	0.00035	2.457
54	-0.04043	0.00442	- 9.147
56	0.08306	0.00560	14.832
62	-0.02947	0.00450	- 6.548

*"t" values less than t_{crt} and not rejected

$n = 27$ observations

$p = 21$ variables (the 20 included in equation at 20th step
plus one for the constant term)

Degrees of freedom = $n - p = 6$

$H_0 : B_1 = 0$

$H_1 : B_1 \neq 0, \alpha/2 = .025$ (two-tailed test)

$t_{\text{crt}} = \pm 2.447$, with $DF = 6$ and $\alpha/2 = .025$

have been made. These assumptions are all related to the residuals or estimates of the error term ϵ_i contained in each developed model. They are:

1. The residuals are clustered around a rectilinear plane, commonly known as the assumption of linearity.
2. The residuals are uniform in their scatter or homoscedasticity is present.
3. The residuals are statistically independent of each other or there is no serial or autocorrelation.
4. The residuals are normally distributed.

It was recognized that these assumptions did exist and each was graphically or statistically tested to establish its validity within the developed models. If these four assumptions are satisfied, it is then possible to measure the sampling error, the error associated with any given point on the regression plane, of the net regression coefficients. These measures could then be used to make valid statistical inferences about the true regression relationships.

Before applying the developed models, an additional check was made for collinearity or simple correlation between the independent variables. When the independent variables in a multiple regression are highly correlated with each other, the net regression coefficients may be unreliable. (2:610) As stated above, these assumptions are related to and tested by the residuals. The residuals are an estimate of the error term ϵ_i commonly expressed as

$$e = Y - Y_c$$

where Y is a specific observed value of the dependent variable, Y_c the estimate of Y calculated by the least squares regression equation $Y_c = a + bX$ and e the residual or deviation of Y from Y_c .

Test of linearity and homoscedasticity. A visual assessment of the plots of residuals against each of the independent variables included in the regression equation is considered to be an adequate and useful check on the validity of the assumptions of linearity and homoscedasticity. (2:608) An examination of Figure 11 indicates that the scatter of Variable 3 plotted against the residuals is approximately uniform and that there is no evidence of curvilinearity. Plots of similar conditions were found to exist throughout all of the variables in the Technique 1 equation. Reference Appendix A. Thus, it was concluded that the assumptions of linearity and homoscedasticity were valid for the Technique 1 model.

Test of statistical independence. When dealing with time series data, there is a distinct possibility that the residuals may not be independent. If they are not and serial correlation can be shown to exist, then the least squares regression analysis may not give the best estimates. The estimates will not contain minimum variance. Yamane recommends the use of the Durbin-Watson test to test whether or not the residuals are statistically independent. A d statistic is figured in terms of deviates and first differences and then compared against critical values prepared by

PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 3 (X-AXIS)

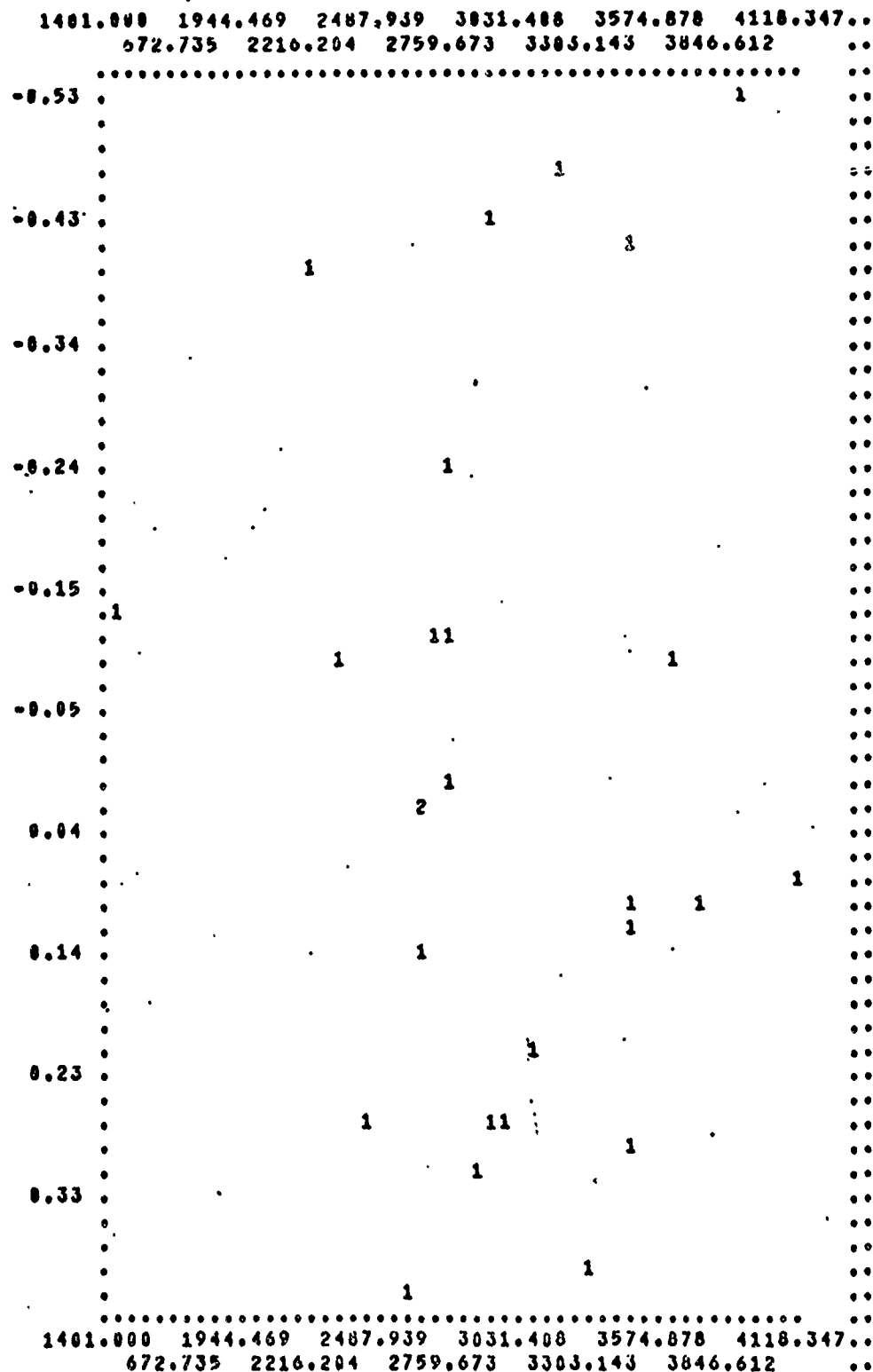


Figure 11

Plot of Residuals Against Variable 3

Durbin and Watson. (24:809-813) Both positive and negative serial correlation is tested by this method. However, it was discovered that the critical value table for d allows for only 5 independent variables. A research of Durbin and Watson's original work in this area indicated that this particular test lost its significance when large numbers of independent variables were present. (11:409-428, 12:159-178) Deprived of this proven test procedure and unable to find a suitable replacement, the assumption of statistical independence among the residuals had to be left untested.

Test of normality. The Fisher g test was employed to test the normality of the residuals obtained at the 20th step of each regression equation. (12:52) The following example illustrates the test performed on the regression equation for Technique 1. The hypothesis was:

$$H_0 : e \sim N (\mu = 0, \sigma^2)$$

$$H_1 : e \not\sim N (\mu = 0, \sigma^2)$$

Two statistics g_1 and g_2 , their variances $V(g_1)$ and $V(g_2)$ and their standardized variates $Z(g_1)$ and $Z(g_2)$ were calculated with the program found in Appendix B. Those values were:

$$g_1 = -0.4437$$

$$g_2 = -0.7729$$

$$V(g_1) = 0.2006$$

$$V(g_2) = 0.7605$$

$$Z(g_1) = -0.9908$$

$$Z(g_2) = -0.8862$$

The algebraic statement in each case was,

$$P [g_1 \leq -0.4437 \mid H_0] = P [Z_{\text{crt}} \leq Z(g_1)]$$

$$P [g_2 \leq -0.7729 \mid H_0] = P [Z_{\text{crt}} \leq Z(g_2)]$$

At an alpha level of .05 and conducting a two-tailed test $Z_{\text{crt}} = \pm 1.96$. The standardized variate in both cases did not exceed the Z_{crt} value, therefore, the null hypothesis could not be rejected. However, it can be said that the distribution is generally platykurtic (somewhat flat) and skewed to the left because of the negative sign on g_1 and g_2 . But because the null hypothesis was not rejected, the assumption of normality was considered satisfied.

Check for high multicollinearity. Multicollinearity refers to the presence of correlation between the independent variables of a regression model. As was noted earlier, it is considered good practice to check for its presence before accepting a regression model as reliable for use. In general, the existence of multicollinearity results in the inaccurate estimation of the regression coefficients because of the large sample variances of the coefficient estimators. (15: 149) Thus, the net regression coefficients become unreliable. It should be noted, however, that while collinearity affects the reliability of individual coefficients in the regression, it may not alter the predictive power of the total regression equation. (2:610)

The method employed for checking the seriousness of multicollinearity called for a comparison of the simple correlation (r_{ij}) between pairs of independent variables and

the coefficient of multiple correlation (R). The simple correlation for each and every independent variable considered was available in the correlation matrix of the BMD output. The "rule of thumb" suggested by Klein is that if $r_{ij} > R$, then the multicollinearity which exists is critical and adversely affects the model. (15:154) The validity of the check in this instance was questioned, however, when an r_{ij} of .99 was compared to an R of .9995 and high collinearity was deemed not to be critical. A rational approach would assume that when near perfect correlation exists between several of the independent variables multicollinearity does exist to a "critical" degree. Therefore, this check was considered inconclusive.

Verification summary. In brief summary, the four assumptions associated with multiple regression were tested and the following conclusions reached:

1. The assumptions of linearity and homoscedasticity were determined to be valid.
2. The assumption of statistical independence among the residuals had to be left untested.
3. The assumption of normality among the residuals was considered valid.

In addition, a check for the critical level of high multicollinearity was considered inconclusive.

Validation of the Model

At this point, a decision had to be made whether to change the basic approach and develop a new model or to test

the forecasting ability of the existing model. Although there were strong indications that the model was deficient, it was decided to proceed with the forecasting and evaluate the results.

As described in Chapter 2, two techniques were utilized in obtaining forecasts. Technique 1 made use of a single model to predict for eleven months. Technique 2 varied the model, by moving the data base, to utilize the most current data in making eleven monthly predictions. In this manner, two sets of engine failure forecasts were compiled. These forecasts are tabulated in Table 5. It was becoming obvious that the selected approach was not working and only a feel for accuracy would be required. The remaining emphasis was to be placed on the reasons for failure to accurately forecast. This "feel for accuracy" was obtained by comparing the absolute differences between the model's forecasts and actual failures to the absolute difference between the mean number of failures and actual failures. The results were tabulated in Table 6. Using the mean number of failures for the period July 1968 through September 1970 would have provided a better forecasting tool than the model developed here.

Table 5
ENGINE FAILURE FORECASTS

POINT IN TIME	FLYING HOURS AND SORTIES ESTIMATED FOR:	FORECAST MADE FOR MONTH OF:	TECHNIQUE 1		TECHNIQUE 2		ACTUAL FAILURES
			FORECAST	FORECAST	FORECAST	FORECAST	
1 October 1970	Oct - Nov	November	29.15	29.15	29.15	11	
1 November 1970	Nov - Dec	December	24.31	24.31	32.77	20	
1 December 1970	Dec - Jan	January	38.79	38.79	1.19	18	
1 January 1971	Jan - Feb	February	33.36	33.36	41.16	21	
1 February 1971	Feb - Mar	March	15.59	15.59	8.56	33	
1 March 1971	Mar - Apr	April	9.78	9.78	14.18	29	
1 April 1971	Apr - May	May	28.20	28.20	36.78	18	
1 May 1971	May - Jun	June	30.65	30.65	18.26	34	
1 June 1971	Jun - Jul	July	36.29	36.29	- 8.85	24	
1 July 1971	Jul - Aug	August	34.21	34.21	45.91	25	
1 August 1971	Aug - Sep	September	30.80	30.80	31.67	18	

Table 6

COMPARISON OF ENGINE FAILURE FORECASTS
AND MEAN ENGINE FAILURES TO THE
ACTUAL ENGINE FAILURES

TECHNIQUE 1 FORECAST	TECHNIQUE 2 FORECAST	MEAN ENGINE FAILURES	ACTUAL ENGINE FAILURES	ABSOLUTE DIFFERENCE		
				TECH. 1 vs ACTUAL	TECH. 2 vs ACTUAL	MEAN vs ACTUAL
29.15	29.15	28.70	11	18.15	18.15	17.70*
24.31	32.77	28.70	20	4.31*	12.77	8.70
38.79	1.19	28.70	18	20.79	16.81	10.70*
33.36	41.16	28.70	21	12.36	20.16	7.70*
15.59	8.56	28.70	33	17.41	24.44	4.30*
9.78	14.18	28.70	29	19.22	14.82	0.30*
28.20	36.78	28.70	18	10.20*	18.78	10.70
30.65	18.26	28.70	34	3.35*	15.74	5.30
36.29	- 8.85	28.70	24	12.29	32.85	4.70*
34.21	45.91	28.70	25	9.21	20.91	3.70*
30.80	31.67	28.70	18	12.80	13.67	10.70*

*Lowest Absolute Difference (per month)

Chapter 4

CONCLUSIONS AND RECOMMENDATIONS

This research effort endeavored to show that TF33-3 engine failures are dependent upon some combination of historical flying hour and sortie data. A method of arrangement was undertaken which transformed the data collected on the B-52H aircraft fleet into specific historical groupings. These specific groups of data were then statistically analyzed and a forecasting model developed. The statistical analysis was performed by the application of multiple correlation and regression techniques to the data. Then monthly forecasts were made for a period of eleven months. This chapter interprets the model behavior, draws conclusions, and makes recommendations for further study.

Interpretation of the Model Behavior

The following relationships were found to exist in the basic data:

1. It was determined by the least squares method that engine failures and flying hours were linear in their relationship. A coefficient of determination (R^2) of 0.3137 indicated that the curve fit was not relatively powerful.
2. A least squares linear relationship was also found to exist between engine failures and sorties. Again,

a low R^2 of 0.3099 indicated a relatively weak curve fit.

3. The best curve fit of sorties versus flying hours was determined to be a curvilinear function with an R^2 of 0.7830.

The population net regression coefficients for each model were tested for significance in two separate manners because of the excessive number of variables present in each model equation. First, the overall significance of each model's regression plane was tested with an F test and in each of the tests it was determined that regression was present in the population and the improvement obtained by fitting these regression planes was not due to chance. Secondly, a t test was applied to each of the population net regression coefficients to test their individual significance. These tests were unsuccessful at the .05 significance level in limiting any model equation to less than eighteen significant variables.

Of the four basic assumptions which should be verified before a multiple regression developed model can be usefully employed, only three were confirmed. The assumptions of linearity, homoscedasticity and normality of the residuals were determined to be valid. An applicable test could not be found to exist for the test of statistical independence among the residuals. Therefore, the assumption had to be left untested. The tests for "critical" levels of multicollinearity were considered inconclusive. However, when near perfect correlation exists as it does between several of the

independent variables, multicollinearity very likely does exist to a critical degree.

Attempts to forecast engine failures with the developed regression models resulted in predictions with wide variances from the failures which actually occurred. The absolute difference between the forecasted and actual engine failures was in several cases twice the actual failure figure. The mean number of engine failures for the period July 1968 through September 1970 was a better predictor of engine failures than were the regression models. In fact, the mean failure was closer to the actual failure for all but three forecasts (See Table 6).

Conclusions

The research hypothesis--a combination of flying hours and sorties can be utilized to yield accurate jet engine failure forecasts--under test in this thesis could not be accepted as a result of the poor forecasting ability of the developed regression models. Even though the research hypothesis could not be accepted as a result of the findings, the use of sorties and flying hours should not be discounted as determinants of jet engine failures. One of the initial premises was that the inclusion of sorties in developing a forecast model would improve the forecasting ability of that model. This premise should be evaluated further. Of the twenty variables included in the regression model, twelve were based upon historical sortie data.

Additionally, in each case, the independent variable which explained the greatest portion of variation in the dependent variable was the current month's observed sorties. These facts would seem to indicate that sorties do have a very significant impact upon engine failure determination and should be further investigated.

The critically high multicollinearity present in the model was apparently the dominant factor which caused the regression model's failure to accurately predict. The authors believe that this phenomena was introduced as a result of the method used to cumulatively arrange the data input. The multicollinearity present in these 64 "created" variables tended to cause the individual net regression coefficients to become unreliable to a degree high enough to affect the forecasting ability of the model.

Recommendations

As a result of the above findings and conclusions, the following recommendations for further studies are made:

1. The hypothesis used in this thesis should be tested further by using different data arrangement techniques. Specifically, the data should be arranged so that minimum multicollinearity is introduced into the regression model. One practical solution would be to use the individual month's observations as the independent variables.

2. A thorough study of the basic flying hour, sortie and engine failure data should be undertaken to define its behavioral patterns.

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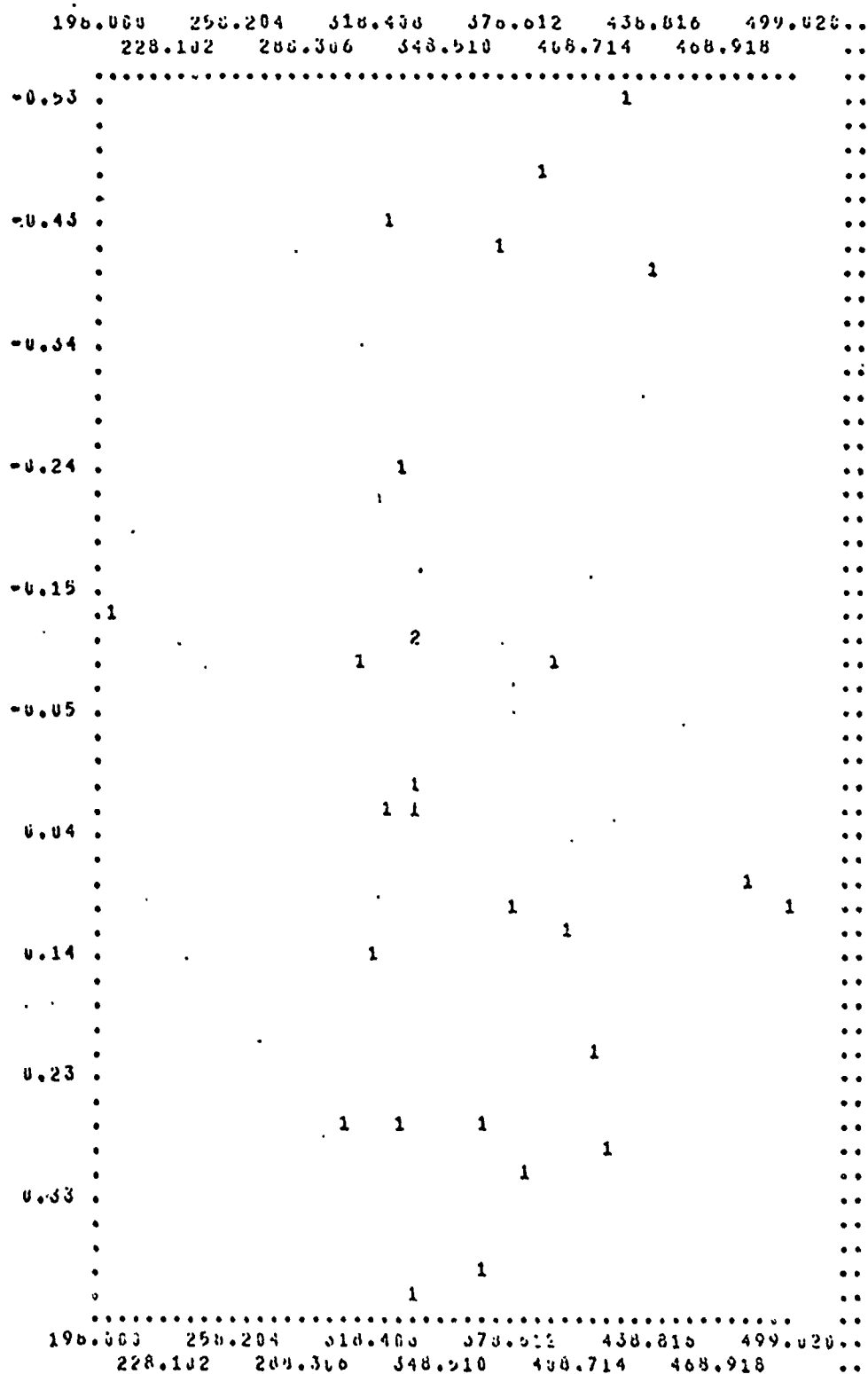
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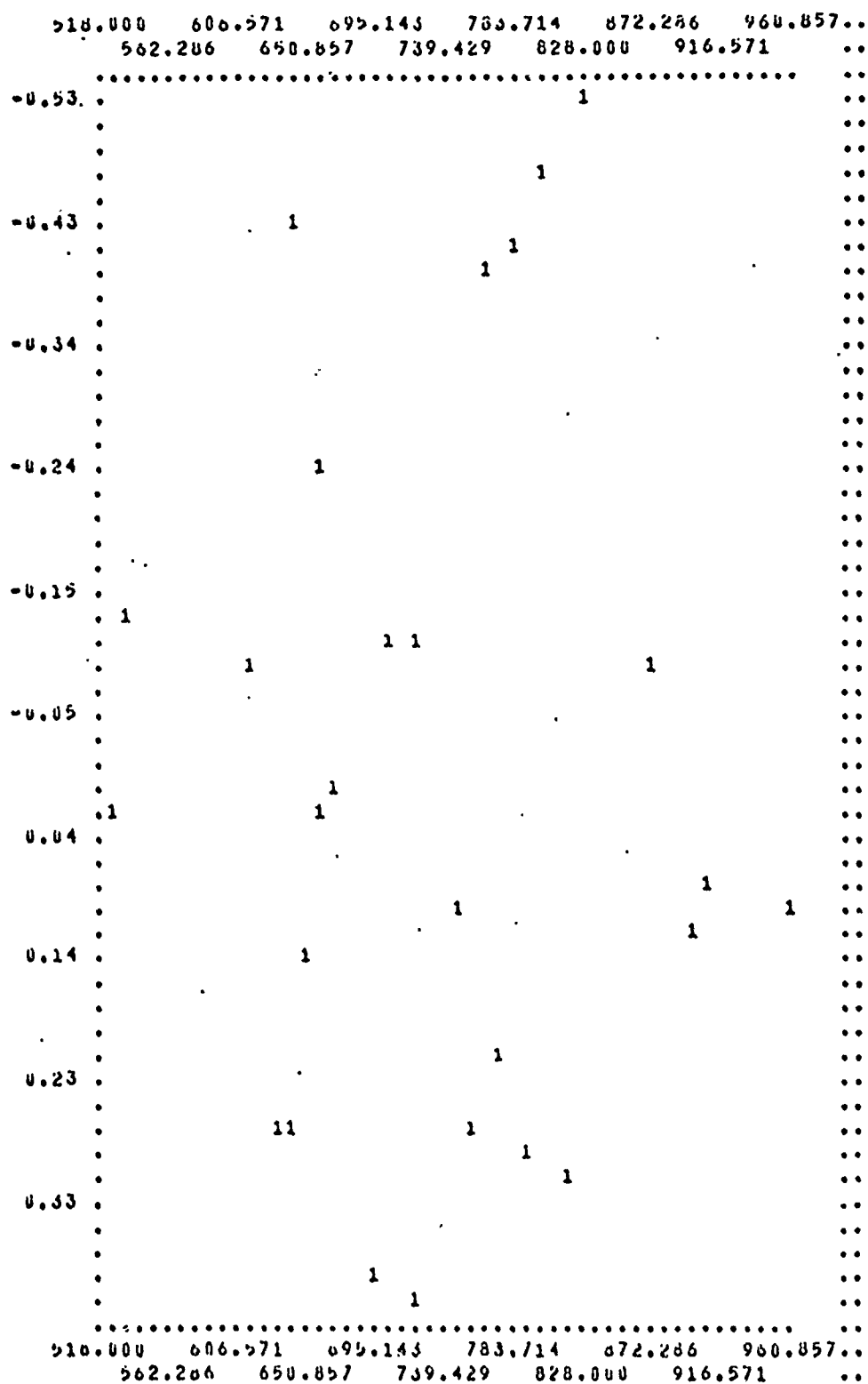
APPENDIX A

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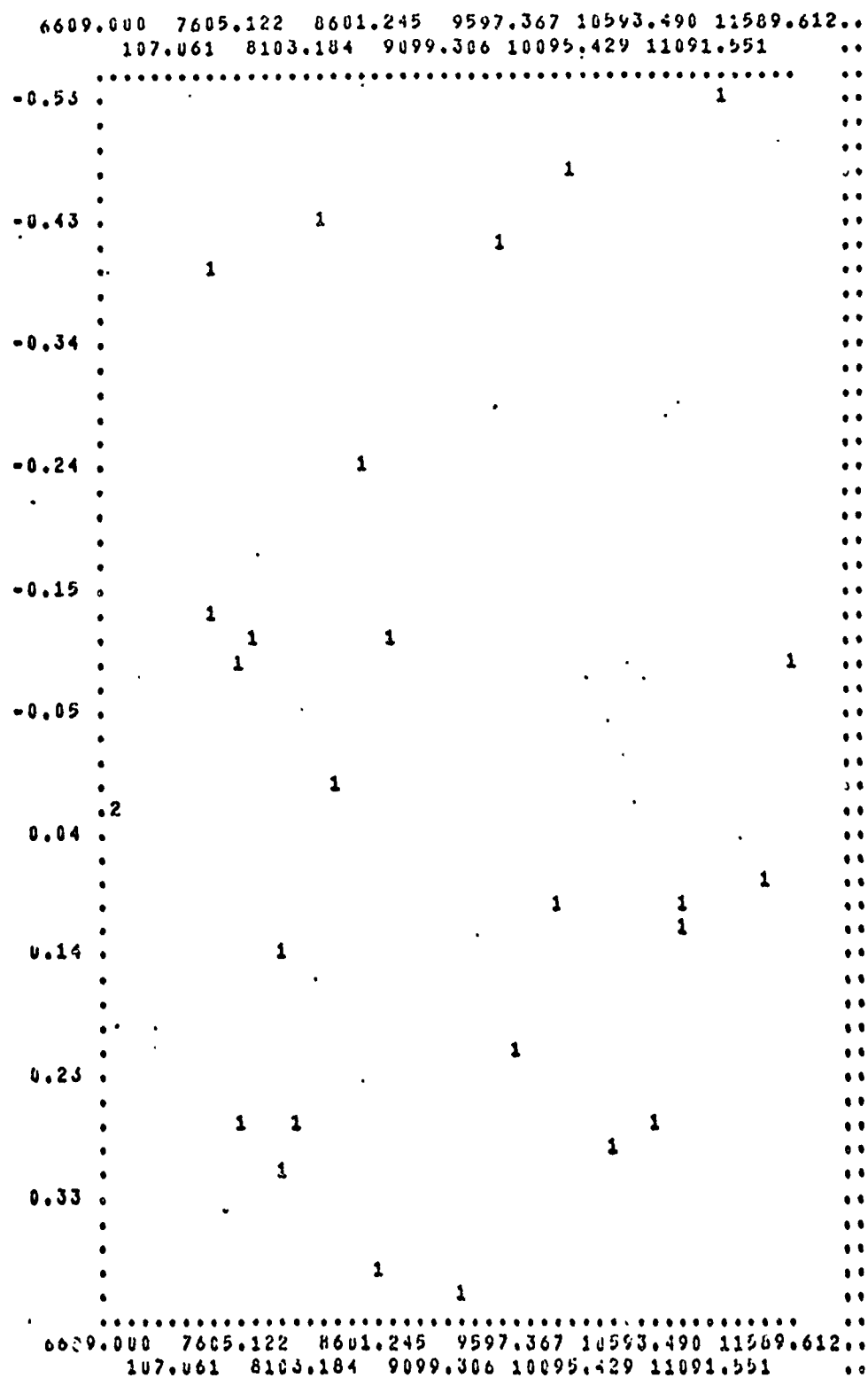
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 2 (X-AXIS)



PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 4 (X-AXIS)

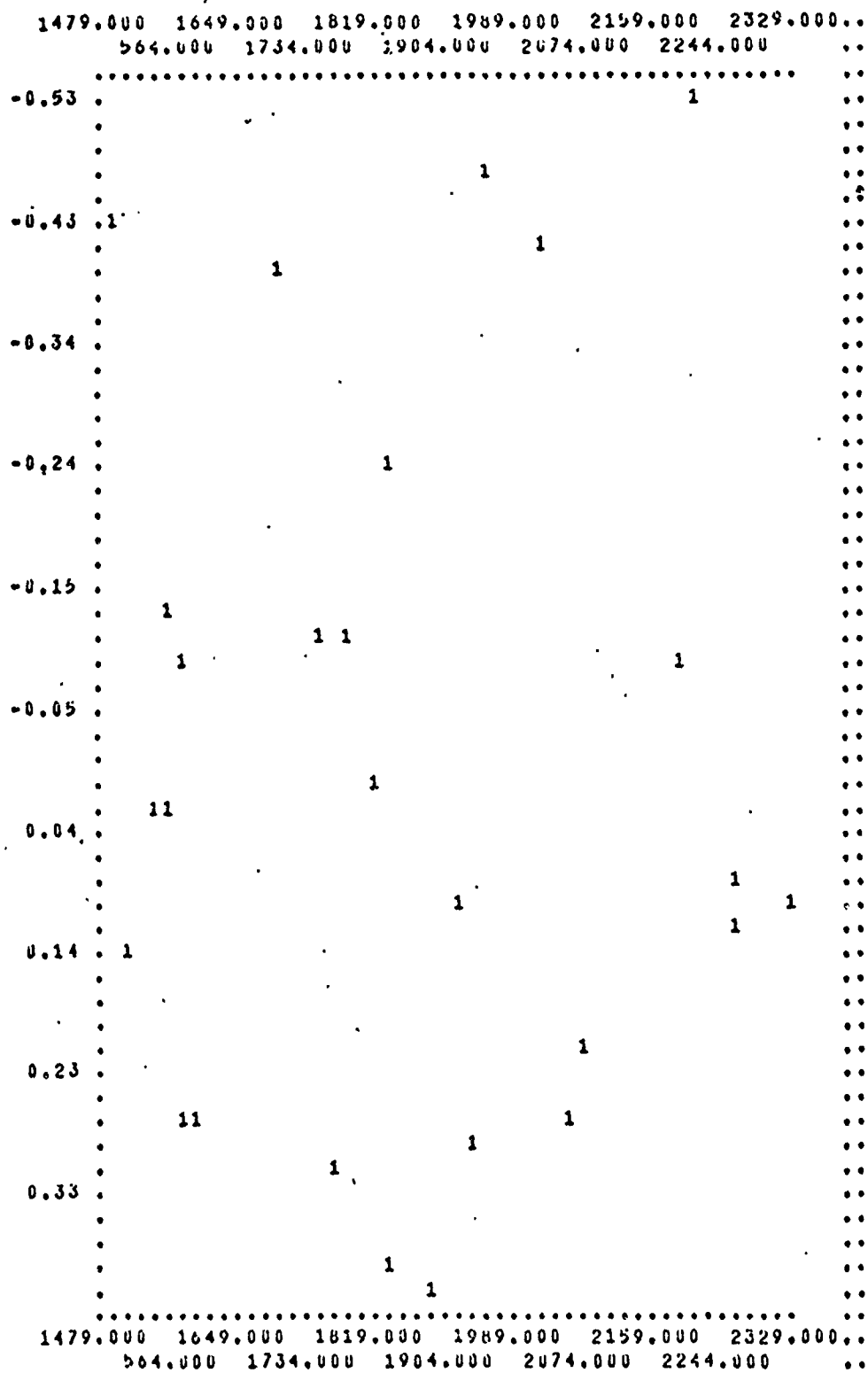


PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 7 (X-AXIS)



PLOT OF RESIDUALS (Y-AXIS)

VS. VARIABLE 10 (X-AXIS)



61

-0.53

-U.43

-4.34

-U.24

-0.15

-0.45

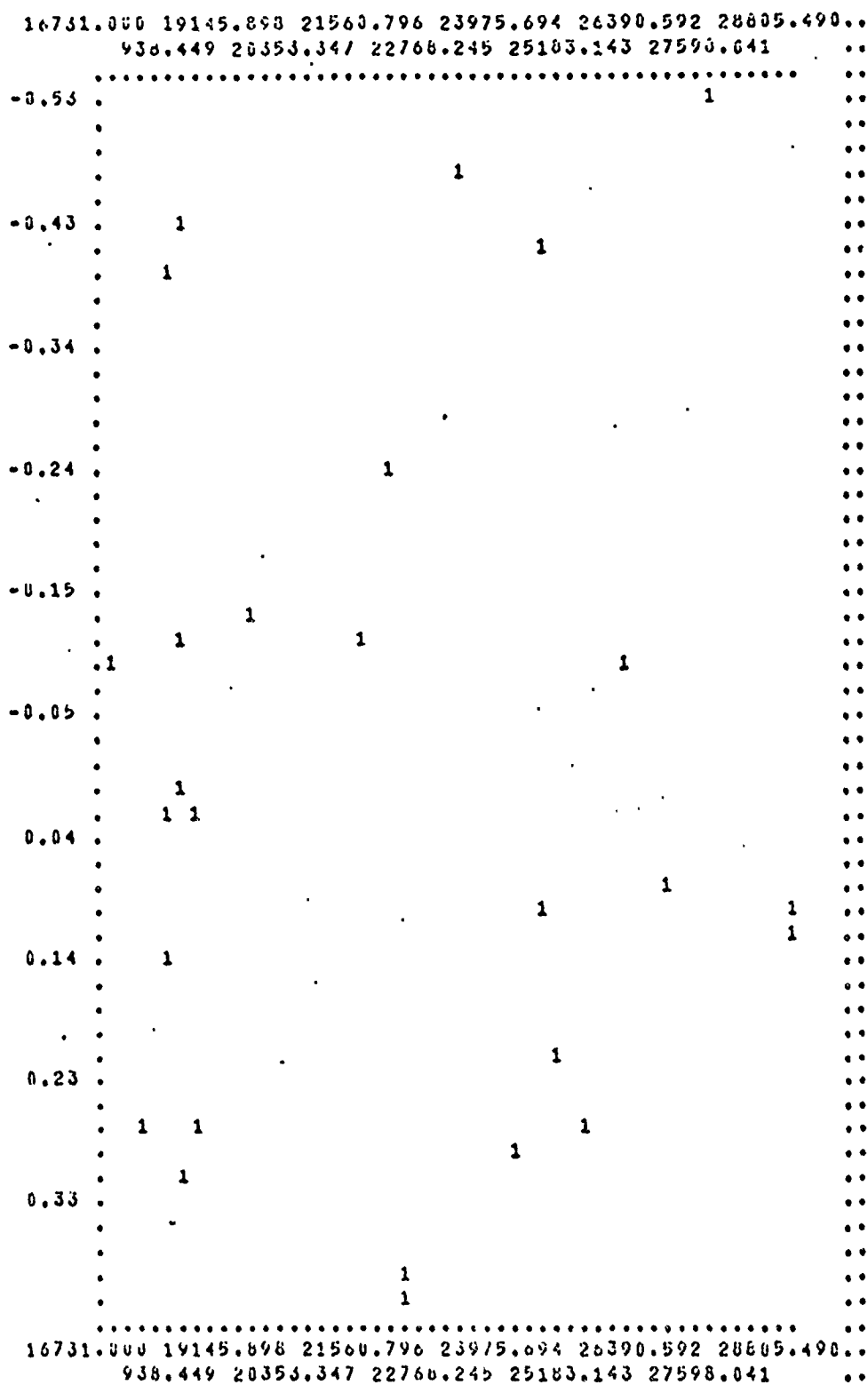
0.04

0.23

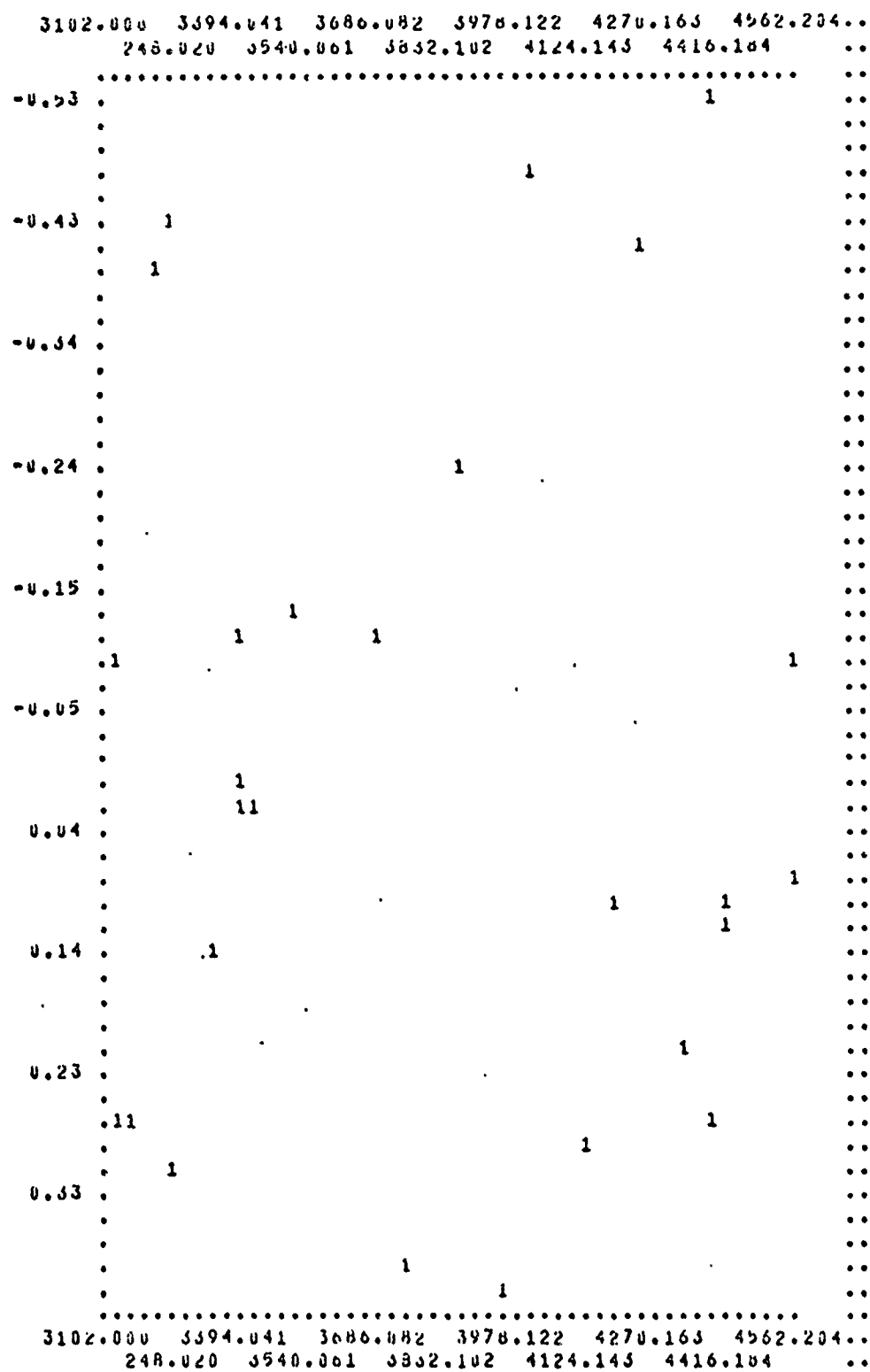
u. 33

1781.000	1998.551	2210.102	2433.053	2651.204	2868.755..
869.776	2107.327	2324.878	2542.429	2759.980	..

PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 15 (X-AXIS)

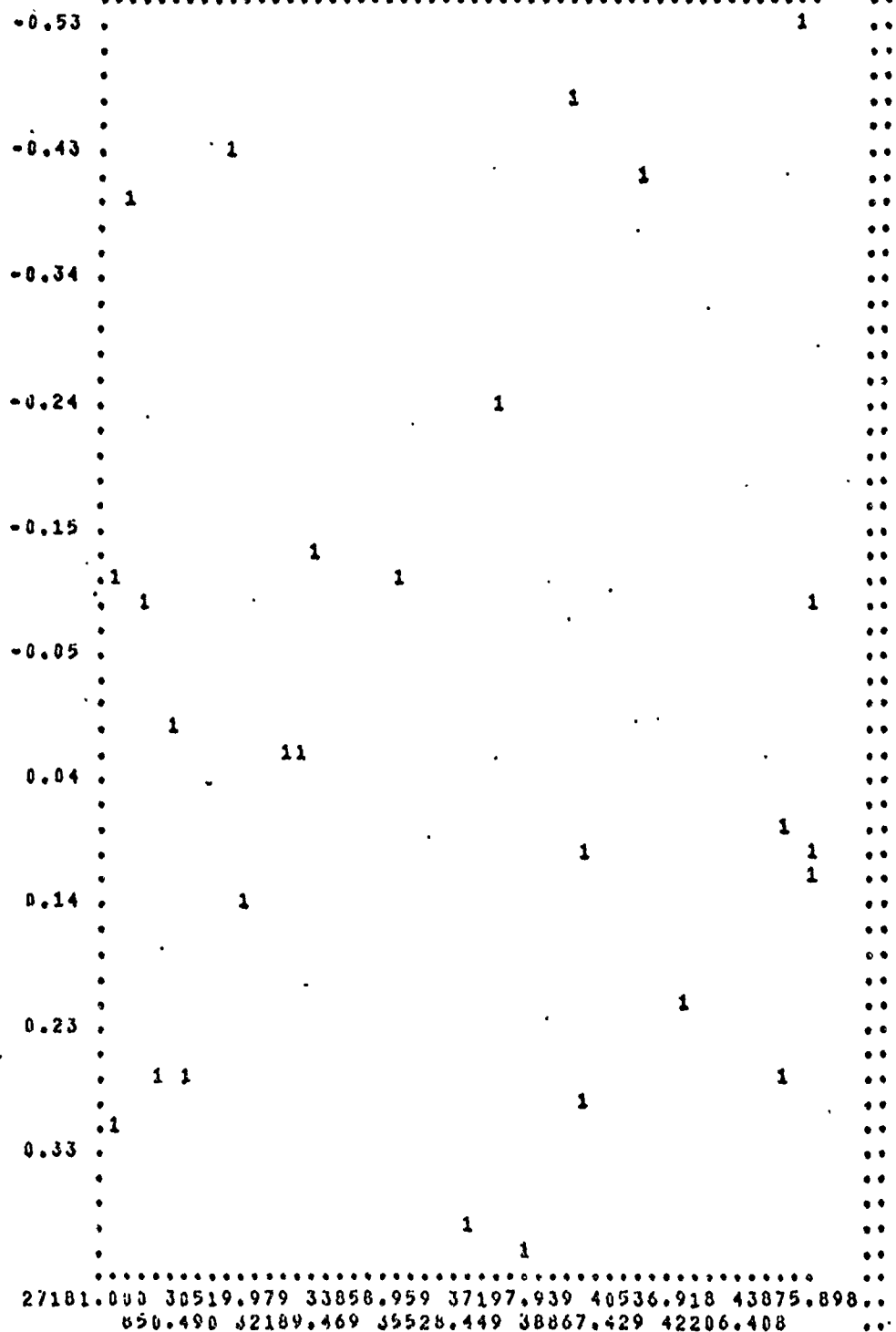


PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 20 (X-AXIS)

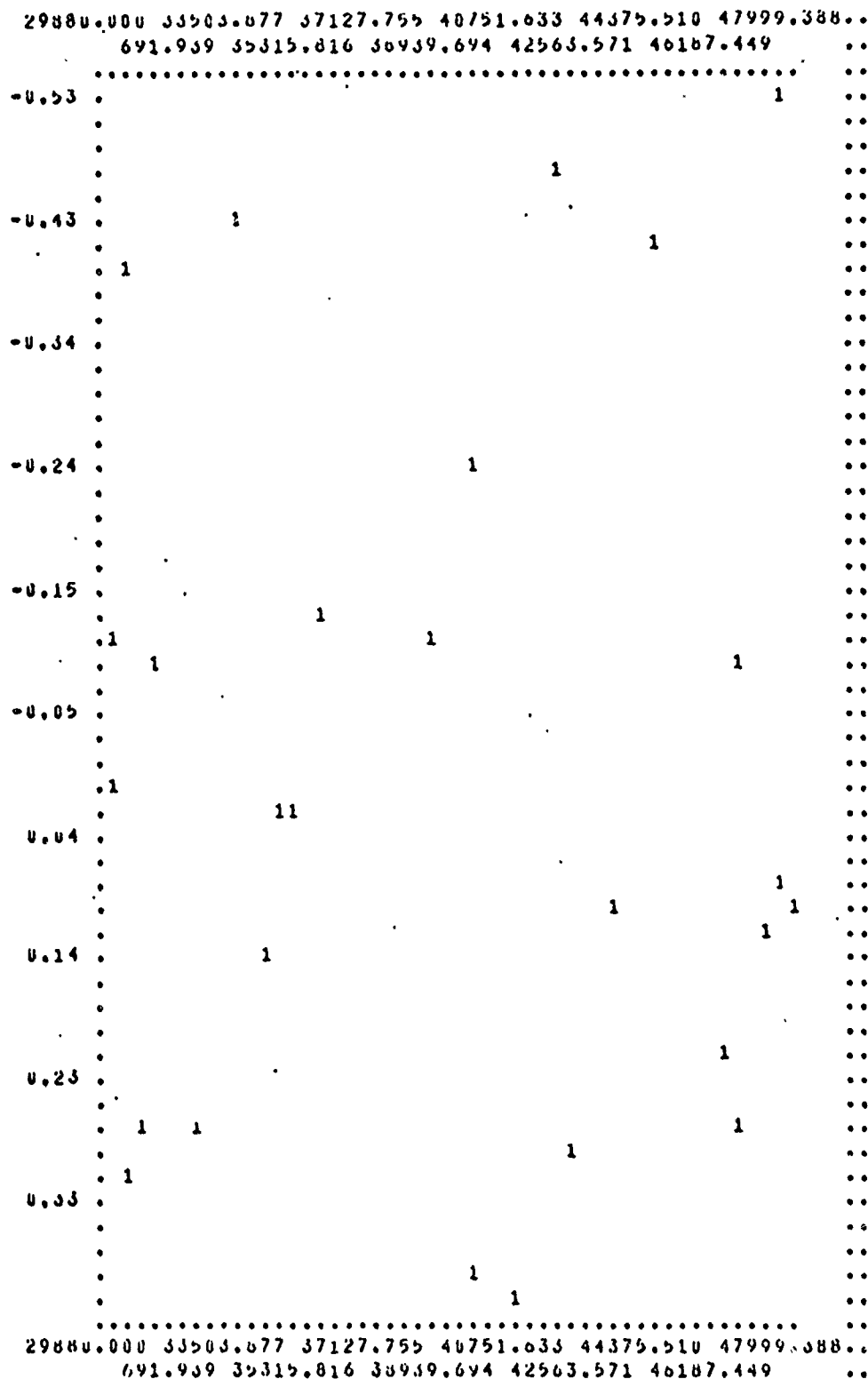


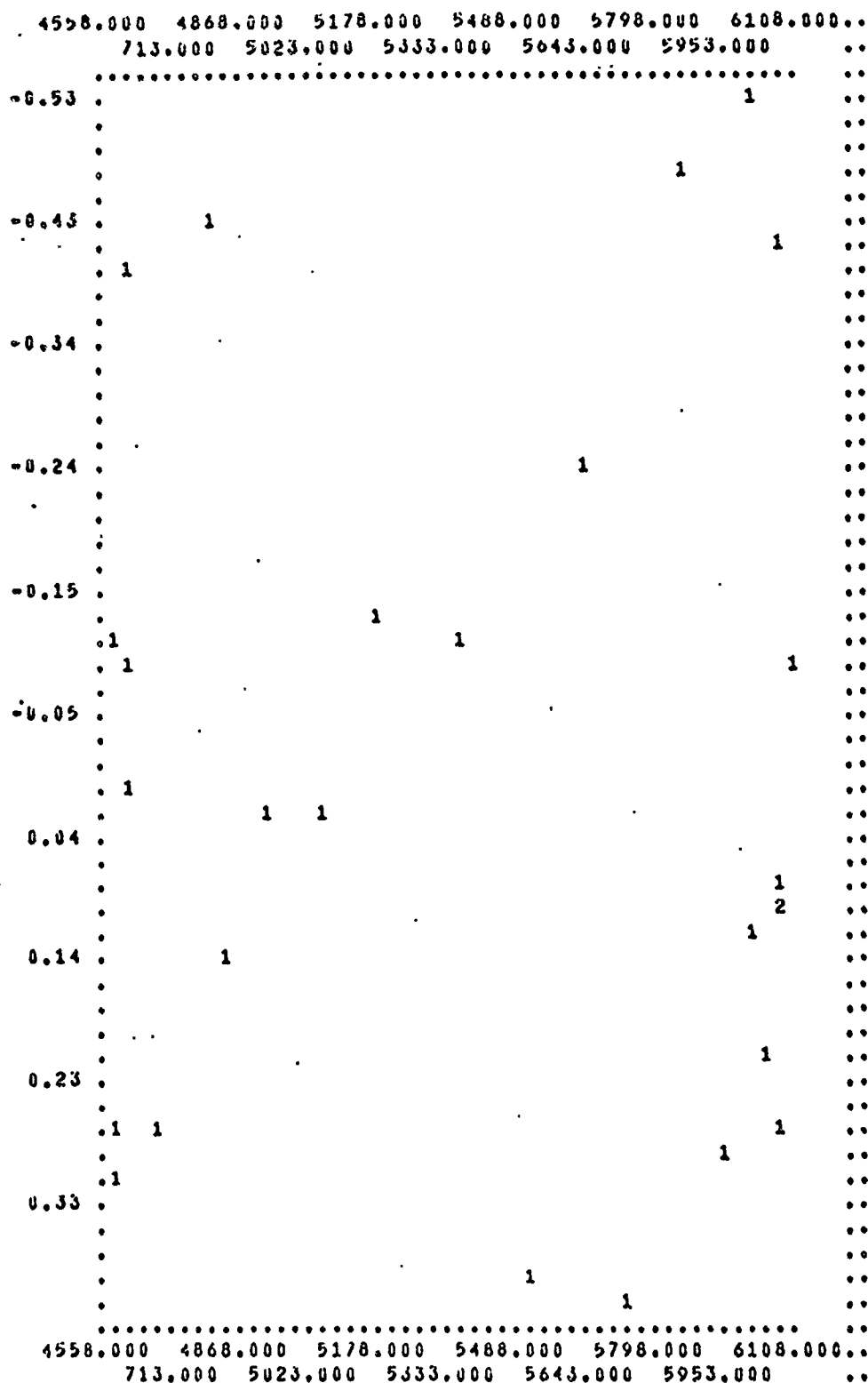
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 23 (X-AXIS)

27181.000 30519.979 33858.959 37197.939 40536.918 43875.898..
850.490 32189.469 35528.449 38867.429 42206.408 ..

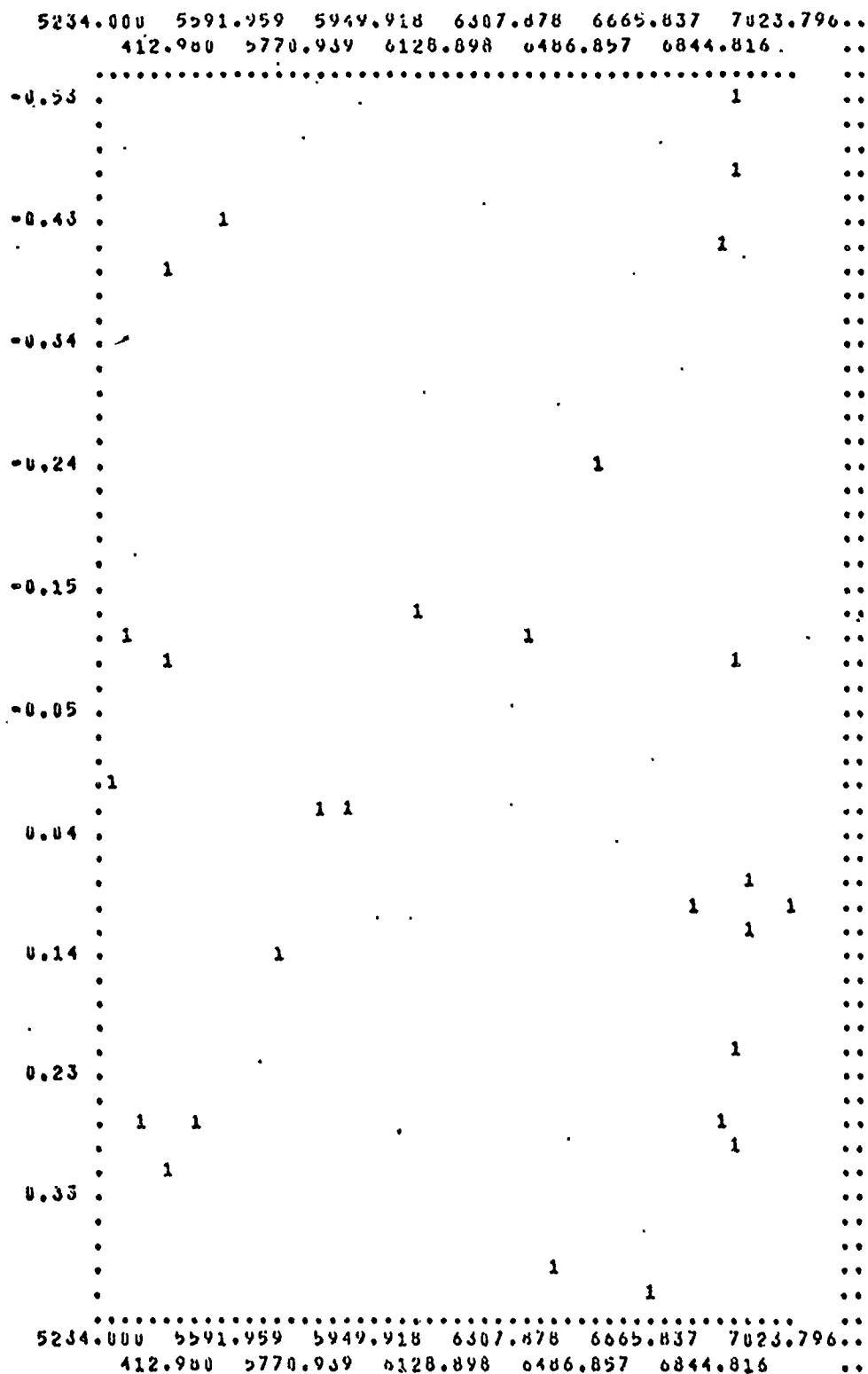


PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 25 (X-AXIS)

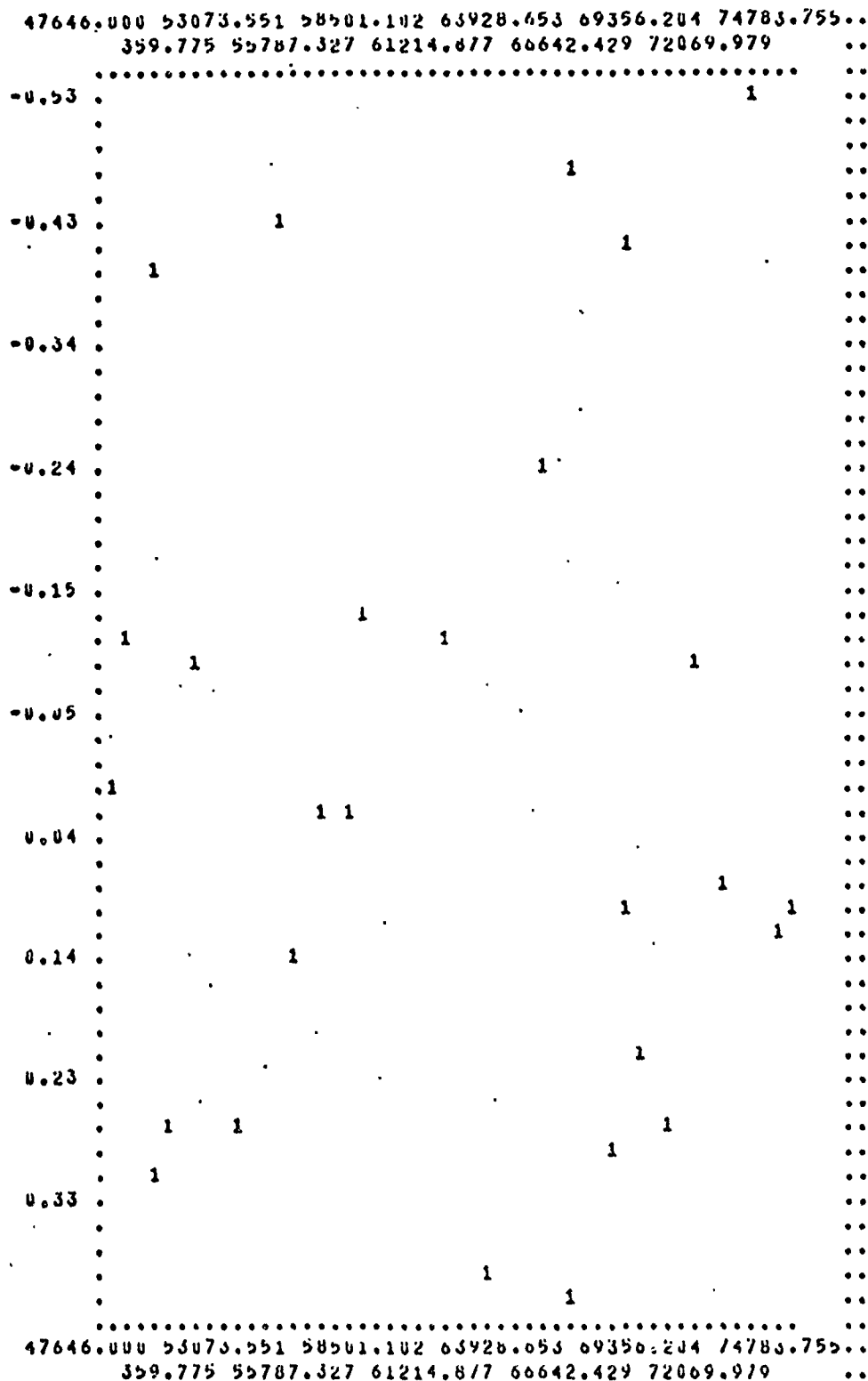




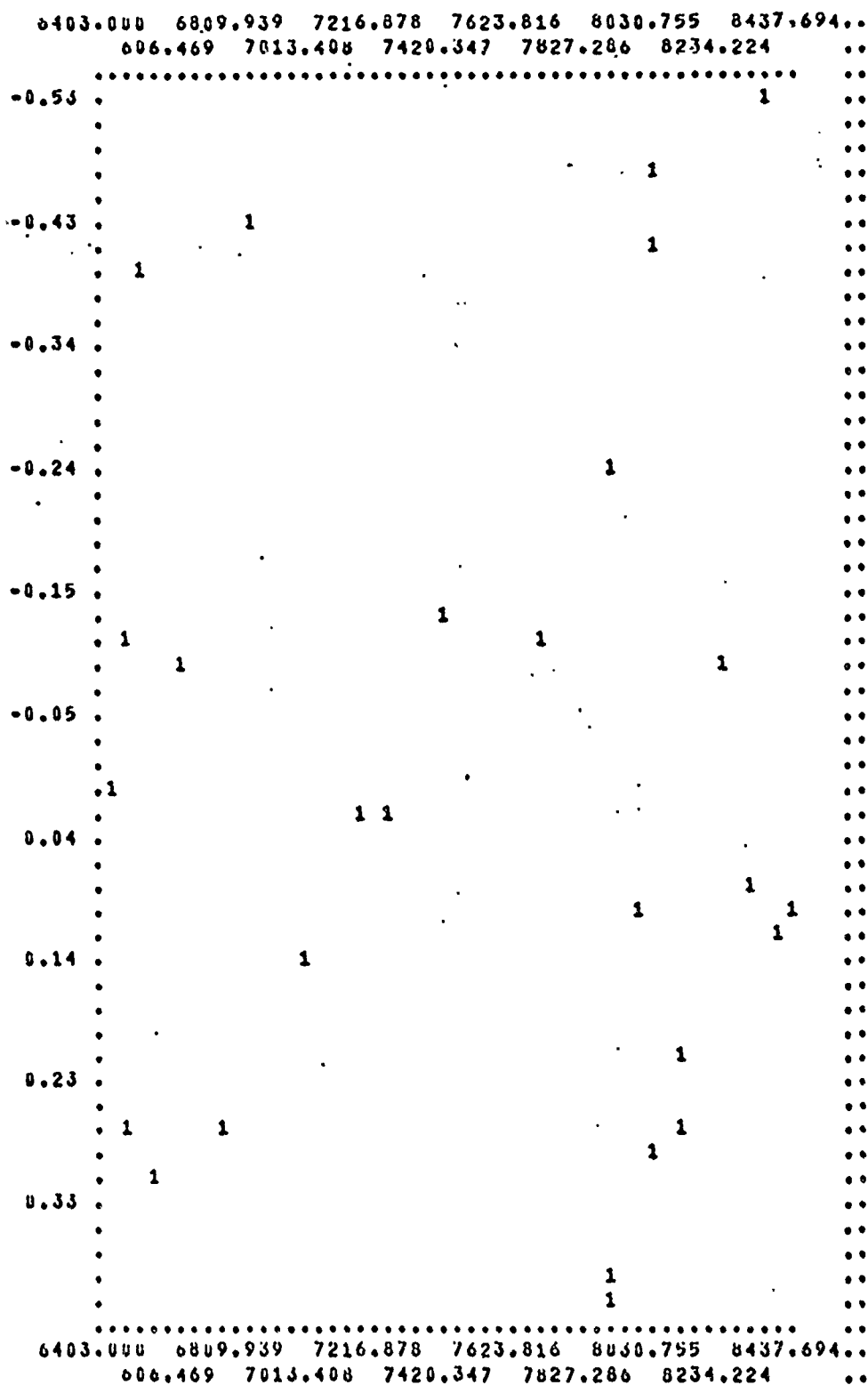
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE J2 (X-AXIS)



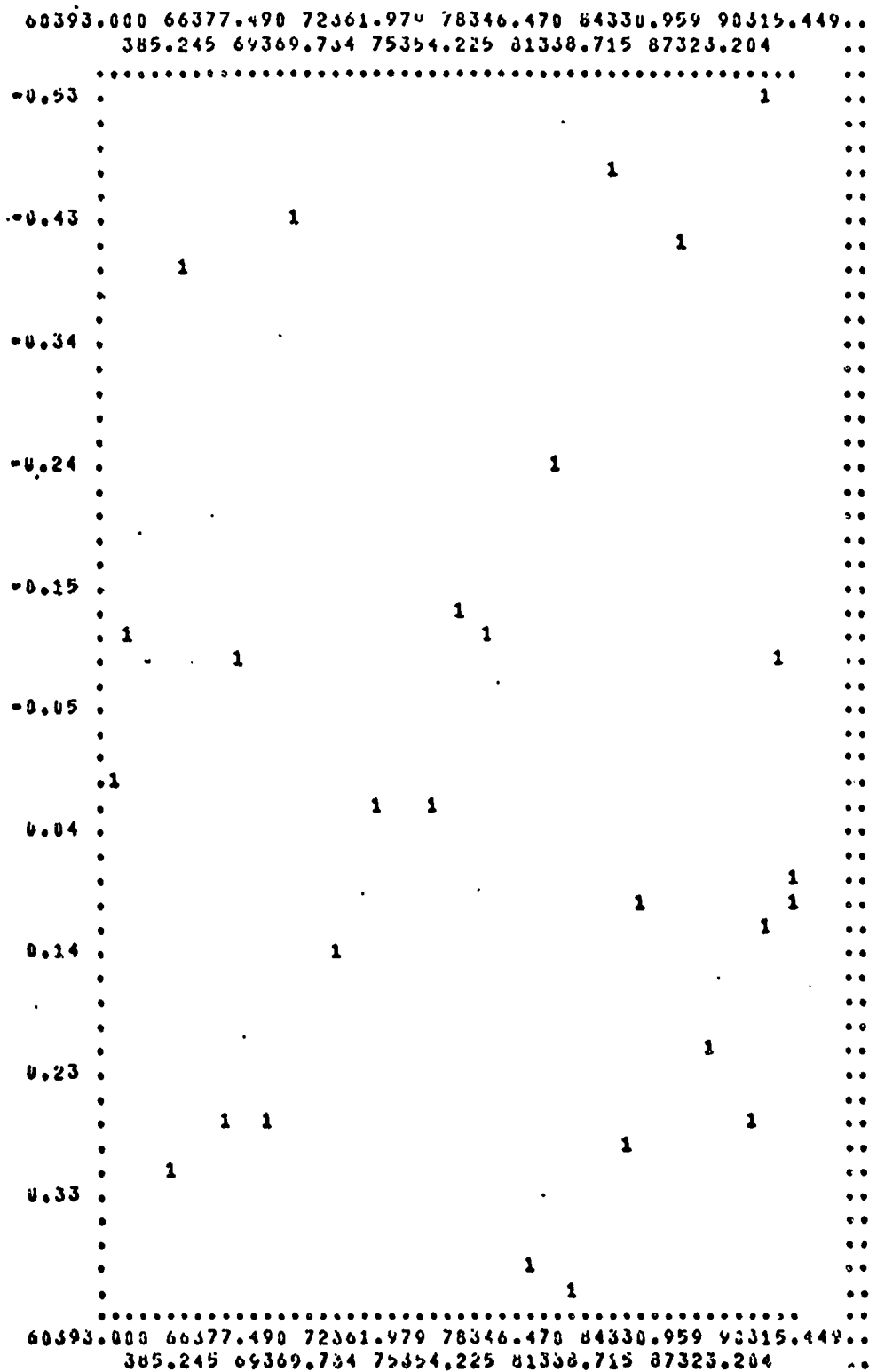
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 37 (X-AXIS)



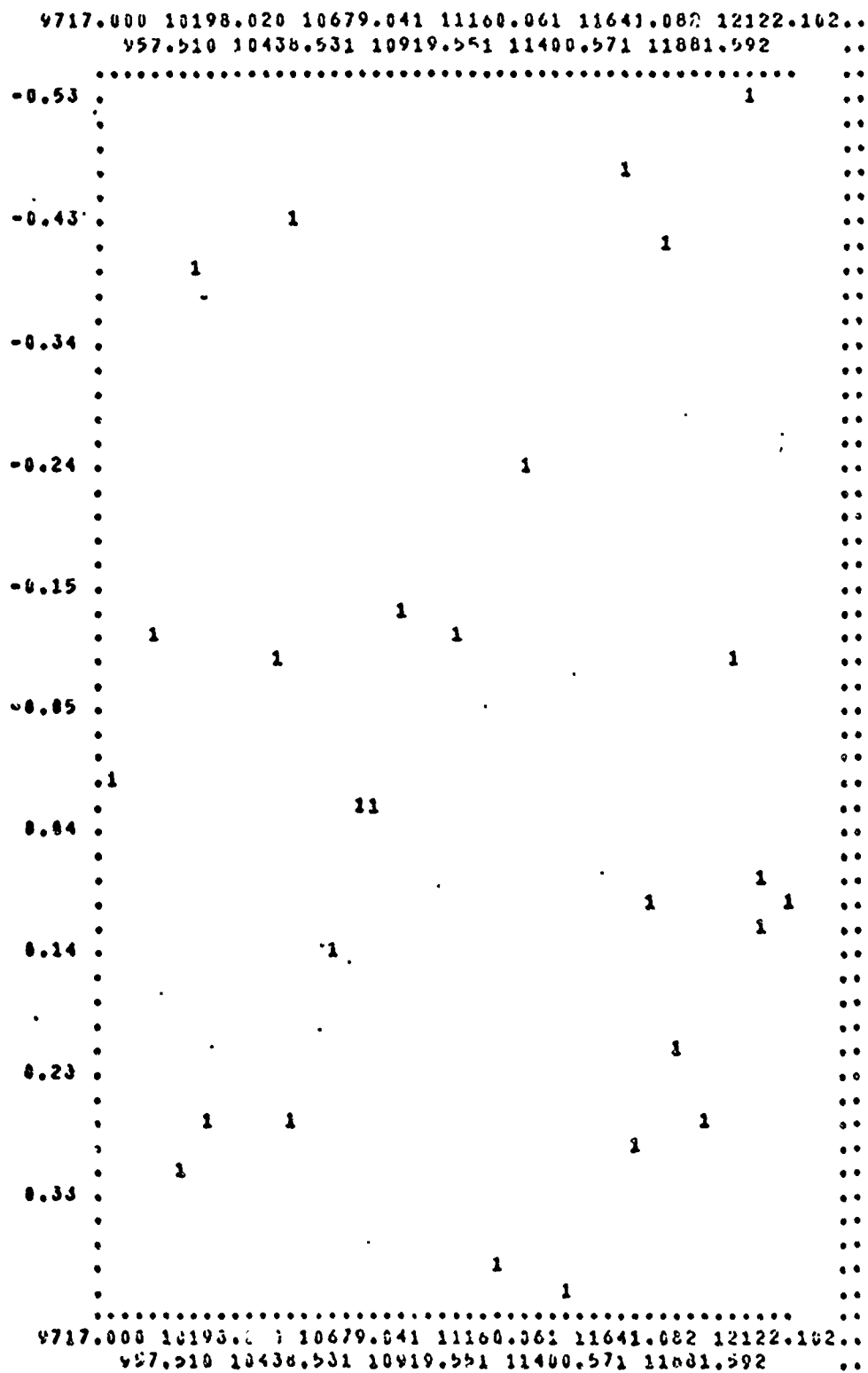
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 38 (X-AXIS)



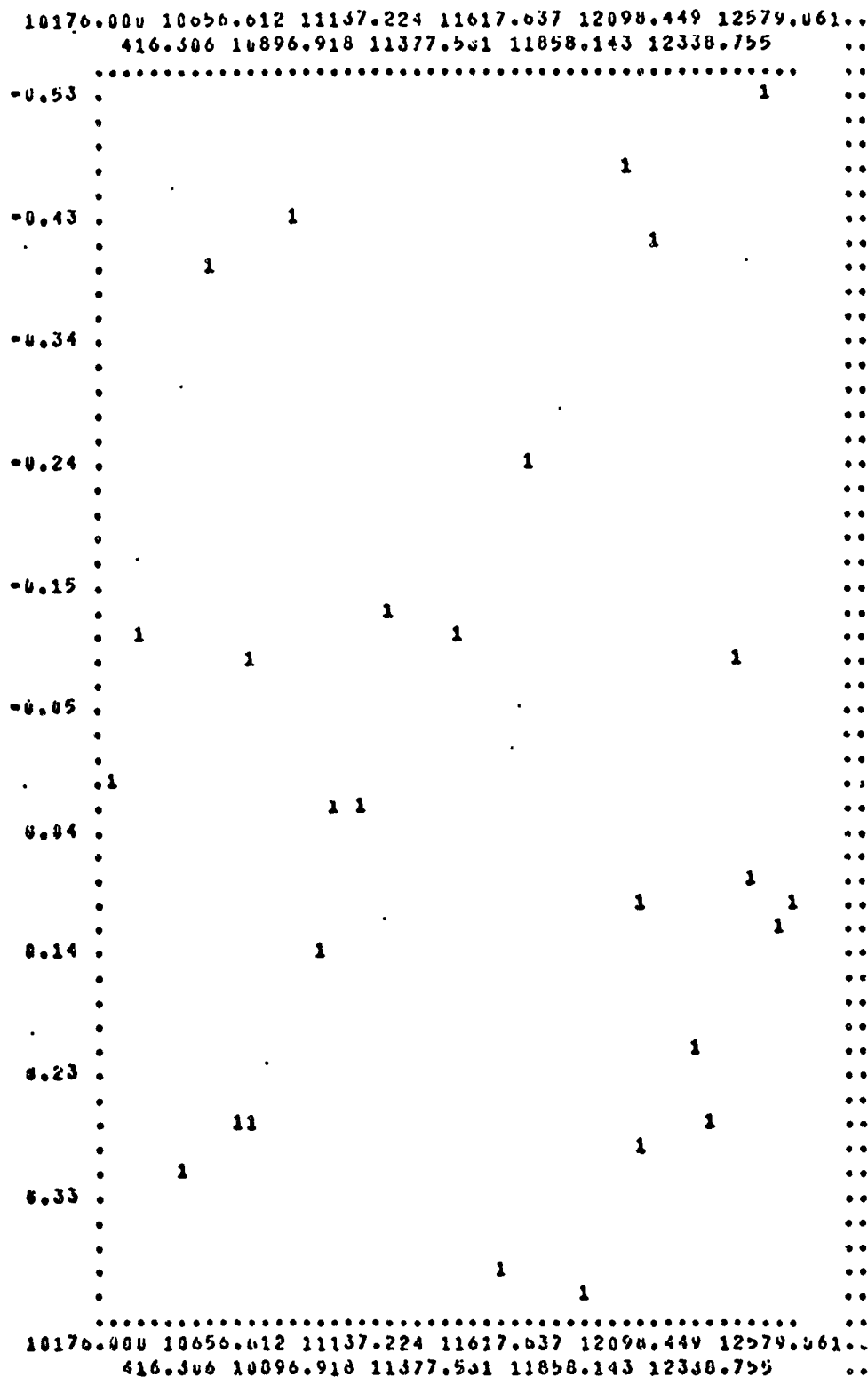
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 45 (X-AXIS)



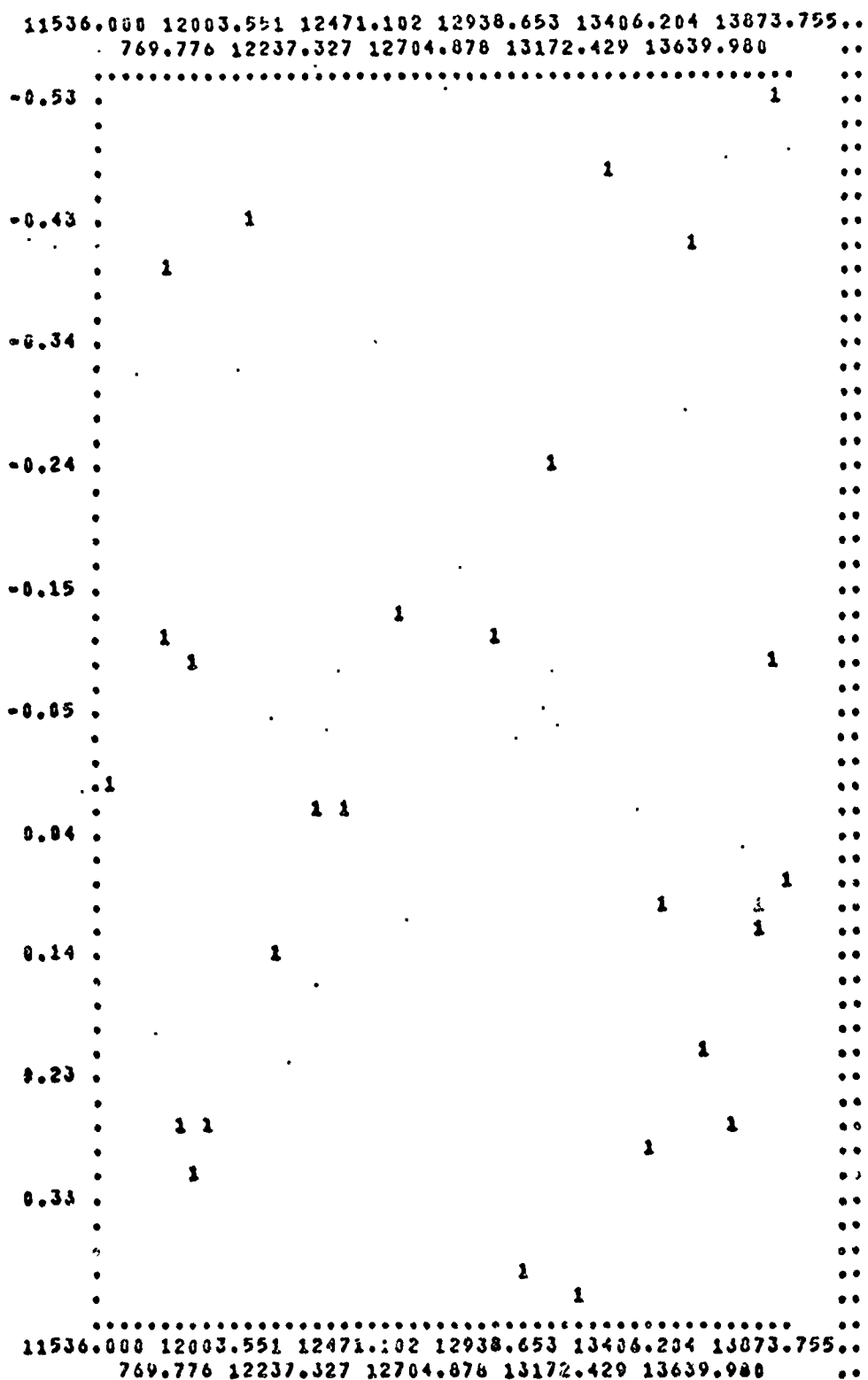
PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 54 (X-AXIS)



PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 36 (X-AXIS)



PLOT OF RESIDUALS (Y-AXIS)
VS. VARIABLE 62 (X-AXIS)



APPENDIX B

15

```

010 DIMENSION X(27),S(4)
020 READ*(X(I),I=1,27)
030 DO 10 I=1,4
040 10 S(I)=0.0
050 DO 20 I=1,27
060 S(1)=S(1)+X(I)
070 S(2)=S(2)+X(I)**2
080 S(3)=S(3)+X(I)**3
090 S(4)=S(4)+X(I)**4
100 XBAR=S(1)/27.
110 PHI2=S(2)-S(1)**2/27.
120 PHI3=S(3)-((3.*S(1)*S(2))/27.)+(2.*S(1)**3)/729.
130 PHI4A=S(4)-(4.*S(1)*S(3))/27.
140 PHI4B=(6.*S(1)**2*S(2))/27.**2-(3.*S(1)**4)/27.**3
150 PHI4=PHI4A+PHI4B
160 S3=((PHI2/25.))**.5)**3
170 S4=(PHI2/25.))**.2
180 G1=(27.*PHI3)/(26.**25.*S3)
190 G2=((27.*(28.*PHI4-(3.*26.*PHI2**2)/27.))/(26.**25.*24.))/S4
200 VC1=(6.**27.**26.)/(25.**28.**30.)
210 VG2=(24.**27.**26.**2)/(24.**25.**30.**32.)
220 ZG1=G1/SQRT(VG1)
230 ZG2=G2/SQRT(VG2)
240 PRINT,"      G1          Z(G1)"
250 PRINT:G1,VG1,ZG1
260 PRINT,"      G2          Z(G2)"
270 PRINT:G2,VG2,ZG2
280 END

```

Appendix B

FORTRAN IV Program for Fisher's g Test of Normality

AFIT
WRIGHT-PATTERSON AFB. OH 45433
UNITED STATES AIR FORCE
OFFICIAL BUSINESS

POSTAGE AND FEES PAID
DEPARTMENT OF THE AIR FORCE

AFIT/SLGR
Wright-Patterson AFB
OHIO 45433

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The student research conducted at this school builds, to a great extent, upon previous research efforts. Thus, your comments and/or criticism regarding this report are earnestly solicited. Please fold the completed sheet so the return address is visible for mailing. COMMENTS:

Tear Out

- Fold - - - - -

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